The Higgs particles in the MSSM

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Abstract

In these lectures, I review the Higgs sector of the Minimal Supersymmetric extension of the Standard Model (MSSM). In a first part, I discuss the link between ultraviolet divergences and symmetries and briefly introduce supersymmetry and the MSSM. In section 2, the two–doublet MSSM Higgs sector will be presented and the Higgs masses and interactions will be derived. In a third part I will discuss the decay modes of the various Higgs bosons, their production at LEP2 and the main experimental constraints on the Higgs spectrum. In section 4, I discuss the prospects for discovering the Higgs particles at the upgraded Tevatron, the LHC and at a high–energy e^+e^- linear collider, and summarize the possibility of studying their properties.

Extended write–up of the Lectures given at the
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1. Divergences, symmetries and Supersymmetry

In this section, we discuss the connexion between ultraviolet divergences and symmetries and present the main argument for introducing low energy Supersymmetry in the description of electroweak interactions: the cancellation of quadratic divergences to the Higgs boson mass. In a first part, we will summarize the electroweak symmetry breaking in a simple abelian model [for more details see the lectures of R. Cahn [1] at this school] and present the basic rules for calculating Feynman diagrams [2], which we will need later. In a second part, and after giving the rules for the calculation of loop diagrams, we discuss the radiative corrections to the electron and photon self–energies in this model, and then analyze the radiative corrections to the Higgs boson mass in the presence of only standard particles and when additional scalar fields are introduced. We finally present the basic features of Supersymmetry and the minimal supersymmetric extension of the Standard Model [3].

1.1 Lagrangians and interactions

In standard QED, the Lagrangian for a fermion of charge e and mass m:

$$\mathcal{L}_{\rm QED} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + i\overline{\psi}\gamma^\mu\partial_\mu\psi - m\overline{\psi}\psi + e\overline{\psi}\gamma^\mu A_\mu\psi \tag{1.1}$$

with A_μ and $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$, the electromagnetic field and tensor. The U(1) gauge invariance, i.e. the invariance of the Lagrangian under the local transformations

$$\psi(x) \to e^{i\alpha(x)}\psi(x)\ ,\ \ A_\mu(x) \to A_\mu(x) - \frac{1}{e}\partial_\mu\alpha(x) \tag{1.2}$$

implies that there is no $A_\mu A^\mu$ term in the Lagrangian so that the photon is massless.

Let us now consider a complex scalar field coupled to itself and to an electromagnetic field A_μ

$$\mathcal{L}_{\rm scal} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + D_\mu\phi^* D^\mu\phi - V(\phi) \tag{1.3}$$

with D_μ the covariant derivative $D_\mu = \partial_\mu - ieA_\mu$ and with the scalar potential

$$V(\phi) = \mu^2\phi^*\phi + \lambda\ (\phi^*\phi)^2 \tag{1.4}$$

The Lagrangian $\mathcal{L}_{\rm scal}$ is again invariant under the local U(1) transformation:

$$\phi(x) \to e^{i\alpha(x)}\phi(x)\quad ,\quad A_\mu(x) \to A_\mu(x) - \frac{1}{e}\partial_\mu\alpha(x) \tag{1.5}$$

For $\mu^2 > 0$, $\mathcal{L}_{\rm scal}$ is simply the QED Lagrangian for a charged scalar particle of mass μ and with ϕ^4 self–interactions. For $\mu^2 < 0$, the field $\phi(x)$ will acquire a vacuum expectation value and the minimum of the potential V will be at

$$\langle\,\phi\,\rangle = \left(-\frac{\mu^2}{2\lambda}\right)^{1/2} \equiv \frac{v}{\sqrt{2}} \tag{1.6}$$

Expanding the Lagrangian around the vacuum state $\langle\phi\rangle$, one obtains

$$\phi(x) = \frac{1}{\sqrt{2}}[v + \phi_1(x) + i\phi_2(x)] \tag{1.7}$$

The Lagrangian becomes then (up to some interaction terms that we omit for simplicity):

$$\begin{aligned}\mathcal{L} &= -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + (\partial^\mu + ieA^\mu)\phi^*(\partial_\mu - ieA_\mu)\phi - \mu^2\phi^*\phi - \lambda(\phi^*\phi)^2 \\ &= -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}(\partial_\mu\phi_1)^2 + \frac{1}{2}(\partial_\mu\phi_2)^2 - v^2\lambda\phi_1^2 + \frac{1}{2}e^2v^2A_\mu A^\mu - evA_\mu\partial^\mu\phi_2 \end{aligned} \tag{1.8}$$

Three remarks can then be made here: (i) There is a photon mass term in the Lagrangian, $\frac{1}{2}M_A^2 A_\mu A^\mu$ with $M_A = ev = -e\mu^2/\lambda$. (ii) We still have a scalar particle ϕ_1 with a mass $M_{\phi_1}^2 = -2\mu^2$. (iii) Apparently, we have a massless particle ϕ_2, a would–be Goldstone boson.

However, there is a problem. In the beginning, we had four degrees of freedom in the theory (two for the complex scalar field ϕ and two for the massless electromagnetic field A_μ) and now we have apparently five degrees of freedom (one for ϕ_1, one for ϕ_2 and three for the massive photon A_μ). Therefore, there must be a field which is not physical at the end and indeed, in $\mathcal{L}_{\rm scal}$ there is a bilinear term $evA_\mu\partial\phi_2$, and we have to get rid of it. To do so, we notice that at first order, we have for the original field ϕ:

$$\phi = \frac{1}{\sqrt{2}}(v + \phi_1 + i\phi_2) \equiv \frac{1}{\sqrt{2}}(v + \phi_1)e^{i\phi_2(x)/v} \tag{1.9}$$

By using the freedom of gauge transformations and by performing also the transformation, $A_\mu \to A_\mu - (1/ev)\partial_\mu\phi_2(x)$, the $A_\mu\partial^\mu\phi_2$ term (and all ϕ_2 terms) disappear in the Lagrangian. This is called the unitary gauge. The moral of the story is then, that the photon (two degrees of freedom) has absorbed the would–be Goldstone boson (one degree of freedom) and became massive (i.e. with three degrees of freedom): the longitudinal polarization is the Goldstone boson. The U(1) gauge symmetry is no more apparent and we say that it is spontaneously broken. This is the Higgs mechanism, with $\phi_1 \equiv H$ being the Higgs field.

Let us, in addition, couple this field to a fermion f (à la Yukawa):

$$\mathcal{L}_f = -\lambda_f.\,\overline{\psi}\psi\phi \tag{1.10}$$

After SSB, i.e. $\phi \to (v + H)/\sqrt{2}$, the fermion acquires a mass $m_f = \lambda_f v/\sqrt{2}$.

Finally, we introduce two scalar fields ϕ_1 and ϕ_2 with a kinetic Lagrangian

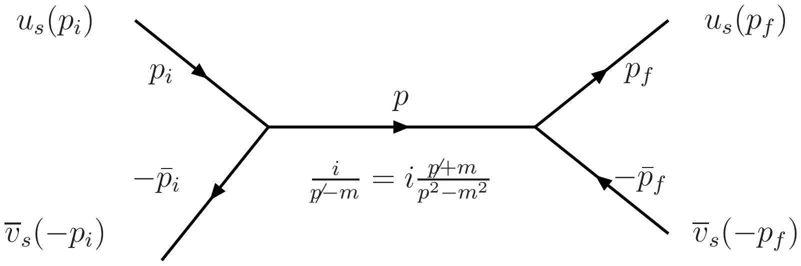
$$\mathcal{L}_{\rm kin} = |\partial_\mu\phi_1|^2 + |\partial_\mu\phi_2|^2 - m_1^2|\phi_1|^2 - m_2^2|\phi_2|^2 \tag{1.11}$$

and with a part which describes the couplings to the scalar field ϕ after SSB (plus, eventually, terms in $\phi_1\phi_2$ that one can omit for simplicity)

$$\mathcal{L}_S = -\lambda_S|H|^2(|\phi_1|^2 + |\phi_2|^2) - 2v\lambda_S H(|\phi_1|^2 + |\phi_2|^2) \tag{1.12}$$

Let us now summarize the basic rules for calculating Feynman diagrams, which we will need in the course of these lectures.

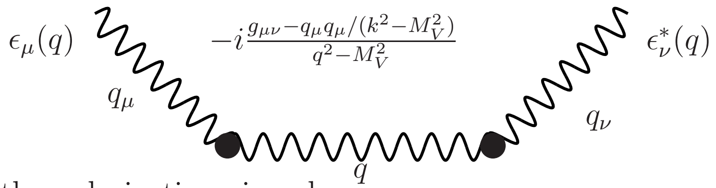
• For each external (anti)–fermion with momentum p and spin s, we associate a spinor. The rules for the spinors and the propagators are summarized in the figure below.



The sum over the spins for the $u\bar{u}$ and $v\bar{v}$ combinations of the spinors gives

$$\Sigma_s\, u_s(p)\, \bar{u}_s(p) = \not{p} + m \;\; , \;\; \Sigma_s\, v_s(p)\, \bar{v}_s(p) = \not{p} - m$$

• For an external gauge boson V_μ with four–momentum q, one associates a polarization vector $\epsilon_\mu(q)$. The rules for the polarization vectors and for the internal propagators (in the unitary gauge) are shown below.



with the sum over the polarization given by

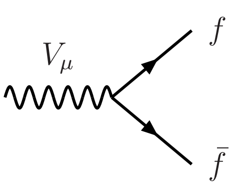
$$\sum_{\text{pol}} = \epsilon^*_\nu \epsilon_\mu = -(g_{\mu\nu} - q_\mu q_\mu/M_V^2)$$

For the photon, one should discard the longitudinal components ($q^\mu q^\nu$) in the expressions above. Note that the transversality of the photon implies: $\epsilon_\mu \cdot q^\mu = 0$.

• For a scalar particle, one simply needs the propagator given by

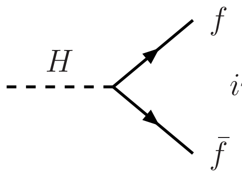
$$- - - - - - - - - - - \qquad i/(q^2 - M_S^2)$$
$$q$$

• The Feynman rule for a given vertex is obtained by multiplying the term involving the interaction in the Lagrangian by a factor $-i$. One needs also to multiply by a factor $n!$ where n is the number of identical particles in the vertex. In the case of the gauge boson couplings to fermions one has:

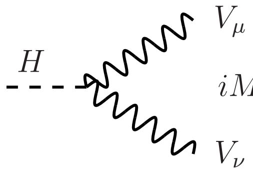


$$ie\gamma_\mu(v_f - a_f\gamma_5)$$
$$Z \,:\, v_f = (2I_f^3 - 4e_f s_W^2)/(4s_W c_W) \;,\; a_f = 2I_f^3/(4s_W c_W)$$
$$W \,:\, v_f = a_f = 1/(2\sqrt{2}s_W)$$
$$\gamma \,:\, v_f = e_f \;,\; a_f = 0$$

The rules for Higgs bosons couplings to fermions and gauge bosons are given by

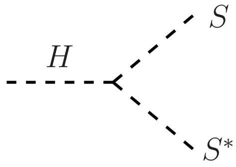


im_f/v

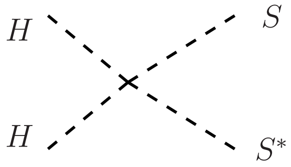


$iM_V^2/v \cdot g_{\mu\nu}$

while the trilinear and quadrilinear Higgs couplings to scalars are:



$iv\lambda_S$



$i\lambda_S$

- The basic relations for the treatment of the Dirac matrices are:

$$\begin{aligned}
&\{\gamma_\mu\,,\,\gamma_\nu\} = \gamma_\mu\gamma_\nu + \gamma_\nu\gamma_\mu = 2g_{\mu\nu} \;\text{ and }\; \not{p} = p_\mu\gamma^\mu \qquad (1.13)\\
&\gamma_5 = (i/4!)\,\epsilon_{\mu\nu\rho\sigma}\gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma = i\gamma^0\gamma^1\gamma^2\gamma^3 \text{ and } \{\gamma_\mu, \gamma_5\} = 0\ ,\ \gamma_5^2 = 1\\
&\mathrm{Tr}(\mathbf{1}) = 4\ ,\ \mathrm{Tr}(\gamma_\mu) = 0\ ,\ \mathrm{Tr}(\gamma_5) = 0\\
&\mathrm{Tr}(A_1A_2) = \mathrm{Tr}(A_2A_1)\ ,\ \mathrm{Tr}(A_1A_2\cdots A_N) = \mathrm{Tr}(A_2\cdots A_NA_1)
\end{aligned}$$

For the contractions of the γ matrices, the rules are:

$$\begin{aligned}
\gamma^\mu\gamma_\mu &= 2g^\mu_\mu + \gamma_\mu\gamma^\nu = g^\mu_\mu = 4 \qquad (1.14)\\
\gamma^\mu\gamma_\nu\gamma_\mu &= \gamma^\mu(2g_{\mu\nu} - \gamma_\mu\gamma_\nu) = 2\gamma_\nu - 4\gamma_\nu) = -2\gamma_\nu\\
\gamma^\mu\gamma^\nu\gamma^\rho\gamma_\mu &= (2g^{\mu\nu} - \gamma^\nu\gamma^\mu)(2g^\rho_\mu - \gamma_\mu\gamma^\rho) = 4g^{\nu\rho} - 2\gamma^\nu\gamma^\rho - 2\gamma^\nu\gamma^\rho + 4\gamma^\nu\gamma^\rho = 4g^{\nu\rho}
\end{aligned}$$

while for the traces of γ matrices, one has

$$\begin{aligned}
\mathrm{Tr}(\gamma^\mu\gamma^\nu) &= \mathrm{Tr}(2g^{\mu\nu} - \gamma^\mu\gamma^\nu) = 2g^{\mu\nu}\mathrm{Tr}(1) - \gamma^\mu\gamma^\nu \Rightarrow \mathrm{Tr}(\gamma^\mu\gamma^\nu) = 4g^{\mu\nu} \qquad (1.15)\\
\mathrm{Tr}(\gamma^{\mu_1}\cdots\gamma^{\mu_n}) &= \mathrm{Tr}(\gamma^{\mu_1}\cdots\gamma^{\mu_{2n+1}}\gamma^5\gamma^5) = (-1)\,\mathrm{Tr}(\gamma^{\mu_1}\cdots\gamma^5\gamma^{\mu_{2n+1}}\gamma^5)\\
&= (-1)^{2n+1}\,\mathrm{Tr}(\gamma^5\gamma^{\mu_1}\cdots\gamma^{\mu_{2n+1}}\gamma^5) = +\,\mathrm{Tr}(\gamma^5\gamma^{\mu_1}\cdots\gamma^{\mu_{2n+1}}\gamma^5) = 0\\
\mathrm{Tr}(\gamma^\mu\gamma_5) &= \mathrm{Tr}(\gamma^\mu\gamma^\nu\gamma^\sigma\gamma_5) = \mathrm{Tr}(\gamma^{\mu_1}\cdots\gamma^{\mu_{2n+1}}\gamma^5) = 0\\
\mathrm{Tr}(\gamma^\mu\gamma^\nu\gamma_5) &= \frac{1}{4}\mathrm{Tr}(\gamma^\alpha\gamma_\alpha\gamma^\mu\gamma^\nu\gamma_5) = (1/4)\,\mathrm{Tr}(\gamma_\alpha\gamma^\mu\gamma^\nu\gamma_5\gamma^\alpha)\\
&= -(1/4)\,\mathrm{Tr}(\gamma_\alpha\gamma^\mu\gamma^\nu\gamma^\alpha\gamma_5) = -\mathrm{Tr}(\gamma^\mu\gamma^\nu\gamma_5) = 0
\end{aligned}$$

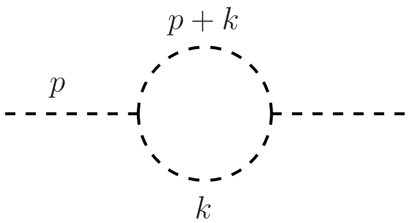
Using the same tricks as above, one can proof that the traces of 4 γ matrices are

$$\begin{aligned}
\mathrm{Tr}(\gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma) &= 4(g^{\mu\nu}g^{\rho\sigma} - g^{\mu\sigma}g^{\nu\rho} + g^{\mu\rho}g^{\nu\sigma}) \qquad (1.16)\\
\mathrm{Tr}(\gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma\gamma_5) &= -4i\epsilon^{\mu\nu\sigma\rho}
\end{aligned}$$

1.2 Loop calculations and the self–energy divergences

1.2.1 Rules for the calculation of loop integrals

Let us first summarize the rules for the calculation of loop integrals, taking the simple example of a two point function with internal scalar particles and external Higgs bosons with momentum p and an HSS^* coupling given by (ig). The Feynman diagram is shown below and corresponds to $-i\Pi(p^2)$ with, if the external particle is on–shell, $p^2 = m^2$. The rules for calculating this diagram are as follows:



$$= -i\Pi(p^2)$$

• The measure of the loop integral over internal momentum k is $\int \mathrm{d}^4k/(2\pi)^4$ (for fermion loops, on needs to take the trace and add a factor (-1) for Fermi statistics). One has then

$$-i\Gamma = (ig)^2 \int \frac{\mathrm{d}^4k}{(2\pi)^4} \frac{i}{(p+k)^2 - m^2} \frac{i}{k^2 - m^2} = g^2 \int \frac{\mathrm{d}^4k}{(2\pi)^4} \frac{1}{(p+k)^2 - m^2} \frac{1}{k^2 - m^2} \quad (1.17)$$

• One needs then to symmeterize the integrand using $1/ab = \int_0^1 \mathrm{d}x/[a + (b-a)x]^2$

$$\Gamma = ig^2 \int \frac{\mathrm{d}^4k}{(2\pi)^4} \int_0^1 \mathrm{d}x \frac{1}{(k^2 + 2pkx + p^2x - m^2)^2} \quad (1.18)$$

• Shift of the variable $k \rightarrow k' = k + px$ (the integrand becomes k^2 symmetric)

$$\Gamma = ig^2 \int \frac{\mathrm{d}^4k}{(2\pi)^4} \int_0^1 \mathrm{d}x \frac{1}{(k^2 + p^2x(1-x) - m^2)^2} \quad (1.19)$$

• Perform the Wick rotation $k_0 \rightarrow ik_0$ to go to Euclidean space ($k^2 \rightarrow -k^2$)

$$\Gamma = -g^2 \int \frac{\mathrm{d}^4k}{(2\pi)^4} \int_0^1 \mathrm{d}x \frac{1}{(k^2 - p^2x(1-x) + m^2)^2} \quad (1.20)$$

• Switch to the polar coordinates for d^4k using: $\int_{-\infty}^{+\infty} d^4k\, F(k^2) = \pi^2 \int_0^\infty dk^2\, k^2\, F(k^2)$

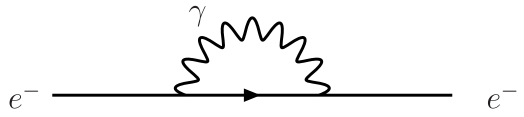
$$\Gamma = -\frac{g^2}{16\pi^2} \int_0^1 \mathrm{dx} \int_0^\infty y\mathrm{d}y \frac{1}{(y - p^2x(1-x) + m^2)^2} \quad (1.21)$$

• Finally, perform the integrals over the variables y and x. If the integral is divergent, make a cut-off at the energy Λ ($\int_0^\Lambda \mathrm{d}y$).

For a scalar particle the contribution of this diagram corresponds to the radiative correction to its squared mass, $\Delta M^2 = -i\Pi(p^2 = M^2)$.

1.2.2 The electron self–energy

The self–energy of the electron of momentum p in QED, $-i\Sigma_e(\not p)$, is given by the diagram



$$\equiv -i\Sigma_e(\not p)$$

Using the rules described in the preceding subsections one obtains for the amplitude:

$$\begin{aligned}
-i\Sigma_e(\not p) &= \int \frac{d^4k}{(2\pi)^4}(-ie\gamma_\mu)\frac{i}{\not p + \not k - m}(-ie\gamma_\nu)\ \frac{-ig^{\mu\nu}}{k^2} \\
&= -e^2 \int \frac{d^4k}{(2\pi)^4}\frac{\gamma_\mu(\not p + \not k + m)\gamma_\mu}{[(p+k)^2 - m^2]k^2} \\
&= -e^2 \int \frac{d^4k}{(2\pi)^4}\frac{-2\,\not p - 2\,\not k + 4m}{[(p+k)^2 - m^2]k^2} \\
&= -e^2 \int_0^1 \mathrm{d}x \int \frac{d^4k}{(2\pi)^4}\frac{-2\,\not p - 2\,\not k + 4m}{(k^2 + 2pkx)^2} \\
&= -ie^2 \int_0^1 \mathrm{d}x \int \frac{d^4k}{(2\pi)^4}\frac{-2\,\not p - 2\,\not k + 2\,\not p x + 4m}{(-k^2 + p^2x^2)^2}
\end{aligned} \tag{1.22}$$

One can then switch to the polar coordinates, integrate over the angular variables and discard the term linear in $\not k$ since the integral is symmetric in k^2

$$\Sigma(\not p) = 2e^2 \int_0^1 \mathrm{d}x \int_0^{+\infty} \frac{\mathrm{d}k^2}{16\pi^2}\frac{2m_f + \not p(-1+x)}{(k^2 + p^2x^2)^2} \tag{1.23}$$

Since the integral is logarithmically divergent one needs to introduce a cut–off Λ. Using the on–shell condition $p^2 = m_e^2$, one has for the radiative correction to the electron mass

$$\delta m_e = \Sigma(\not p)|_{\not p \neq m_e} = \frac{e^2 m_e}{8\pi^2}\int_0^1 \mathrm{d}x \int_0^{\Lambda^2} \mathrm{d}y \frac{y(1+x)}{(y^2 + m_e^2x^2)^2} \tag{1.24}$$

One can then perform the two integrals on y and x which give, respectively:

$$\begin{aligned}
\int_0^1 \mathrm{d}x(1+x)\int_0^{\Lambda^2} \mathrm{d}y \frac{y}{(y + m_e^2x^2} &= \int_0^1 \mathrm{d}x(1+x)\left[\log\frac{\Lambda^2 + m_e^2x^2}{m_e^2x^2} - \frac{\Lambda^2}{\Lambda^2 + m_e^2x^2}\right] \\
&= +\frac{3}{2}\log\frac{\Lambda^2 + m_e^2}{m_e^2} + \frac{\Lambda}{m_e}\arctan\left(\frac{m_e}{\Lambda}\right)
\end{aligned} \tag{1.25}$$

Discarding the terms which vanish for $\Lambda \to \infty$, one finally obtains for the radiative correction to the electron mass in QED:

$$\delta m_e \simeq \frac{3\alpha}{4\pi} m_e \log\frac{\Lambda^2}{m_e^2} + \cdots \tag{1.26}$$

As can be seen there is an ultraviolet logarithmic divergence $\delta m_e \propto \log\Lambda$ for $\Lambda \to \infty$. Since QED is a renormalizable theory, this divergence is removed at all orders by defining the physical mass of the electron, in terms of the bare mass which appears in the Lagrangian to be $m_e^{\rm phys} = m_e^{\rm bare} + \delta m_e$. However, one might view the scale Λ as being the cut–off for which the theory is not valid anymore, i.e. the Grand Unification (GUT) scale $M_{\rm GUT} \sim 10^{16}$ GeV or the Planck scale $M_P \sim 10^{18}$. Since the correction is also proportional to m_e, it is in fact moderate even at these high scales

$$\delta m_e \sim 0.2 m_e \ \text{ for } \Lambda \sim M_P \tag{1.27}$$

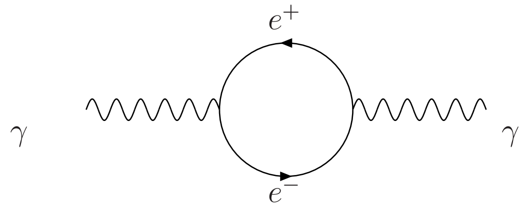
At a more fundamental level, the correction to the electron mass is small because of chiral symmetry: for vanishing electron mass, $m_e = 0$, the QED Lagrangian $\mathcal{L}_{\rm QED}$ is in fact invariant under the chiral transformation

$$\psi_L \to e^{i\theta_L}\psi_L \ \text{ and } \ \psi_R \to e^{i\theta_R}\psi_R \ \text{ with } \ \psi_{L,R} = 1/2(1 \mp \gamma_5)\psi \tag{1.28}$$

However, the fact that $m_e \neq 0$ breaks the chiral symmetry and the radiative correction is proportional to the electron mass. Thus, the chiral symmetry protects the electron mass (and in general any fermion mass) from ultraviolet divergences.

1.2.3 The photon self–energy

In standard QED, the self–energy of the photon with four–momentum p, is given by the following two–point function diagram where electrons are running in the loop:



$\equiv -i\Pi^{\gamma\gamma}_{\mu\nu}(p^2)$

Again, using the rules presented in the previous subsections, one finds for the amplitude:

$$-i\Pi^{\gamma\gamma}_{\mu\nu}(p^2) = \int \frac{d^4k}{(2\pi)^4}(-1){\rm Tr}(-ie\gamma_\mu)\frac{i}{\not k - m}(-ie\gamma_\nu)\frac{i}{\not p + \not k - m} \tag{1.29}$$

$$\Rightarrow \Pi^{\gamma\gamma}_{\mu\nu}(p) = -i\ e^2 \int \frac{d^4k}{(2\pi)^4}\frac{{\rm Tr}\gamma_\mu(\not k + m)\gamma_\nu(\not p + \not k + m)}{[(p+k)^2 - m^2](k^2 - m^2)} \tag{1.30}$$

Using the rules for the Dirac and the loop integral calculations, one has:

$$\begin{aligned}
\text{Denominator} &= \int_0^1 \frac{{\rm d}x}{(k^2 + 2pkx + p^2x - m^2)^2} = \int_0^1 \frac{{\rm d}x}{[(k+px)^2 + p^2x(1-x) - m^2]^2} \\
\text{Numerator} &= {\rm Tr}[\gamma_\mu \not k \gamma_\nu(\not p + \not k) + m^2\gamma_\mu\gamma_\nu] = k^\rho (k+p)^\sigma {\rm Tr}[\gamma_\mu\gamma_\rho\gamma_\nu\gamma_\sigma + m^2\gamma_\mu\gamma_\nu] \\
&= 4[2k_\mu k_\nu + (m^2 - k^2 - p.k)g_{\mu\nu}]
\end{aligned} \tag{1.31}$$

After the shift $k \longrightarrow k+px$, the Wick rotation $k_0 \longrightarrow ik_0$ and the switch to polar coordinates and integration over the angular variables, and the use of the symmetry relation $\int d^4k(k_\mu k_\nu) = \frac{1}{4}g_{\mu\nu}\int d^4k(k^2)$, one obtains:

$$\begin{aligned}\Pi^{\gamma\gamma}_{\mu\nu}(p^2) &= -ie^2 \times 4 \times 2\pi^2 \times \frac{i}{16\pi^4} \times \int_0^1 \mathrm{d}x \int_0^\infty y dy \\ & \frac{[\frac{1}{2}y + m^2 - x(1-x)p^2]g_{\mu\nu} + 2x(1-x)[g_{\mu\nu}p^2 - p_\mu p_\nu]}{[y+m^2-p^2x(1-x)]^2}\end{aligned} \tag{1.32}$$

using the usual tricks and a cut–off Λ for the integral on k^2, one gets

$$\delta m_\gamma = \frac{1}{4}g^{\mu\nu}\,\Pi^{\gamma\gamma}_{\mu\nu}(0) = \frac{e^2}{16\pi^2}\int_0^1 \mathrm{d}x \int_0^{\Lambda^2} dy \frac{y^2+2m^2y}{(y+m^2)^2} \sim \frac{\alpha}{4\pi}\Lambda^2 \tag{1.33}$$

We have a quadratic divergence, while one should have a massless photon, and thus no radiative corrections at all orders because of the $\mathrm{U}(1)_{\mathrm{QED}}$ gauge invariance! The problem is that the use of the cut–off Λ violates the QED gauge invariance. Gauge invariance also tells us that the photon should be transverse, i.e. the self–energy should be proportional to $g_{\mu\nu}p^2 - p_\mu p_\nu$ and therefore the first term, $\propto g_{\mu\nu}$, in the expression above should vanish and we should be left with a contribution of the self–energy of the form

$$\Pi^{\gamma\gamma}_{\mu\nu}(p^2) = \frac{N_c e_f^2 e^2}{2\pi^2}(g_{\mu\nu}p^2 - p_\mu p_\nu)\int_0^1 \mathrm{d}x \int_0^\infty y dy \frac{2x(1-x)}{[y+m^2-p^2x(1-x)]^2} \tag{1.34}$$

The solution is to use a procedure which regulates the ultra–violet divergence while preserving gauge invariance. The dimensional dimensional regularization, where one works in a space–time of $n = 4-\epsilon$ dimensions, fulfills this condition:

– The internal momentum is in n dimensions and the loop measure is $\int d^nk/(2\pi)^n$ etc...

– The Dirac algebra is in n dimensions with rules: $\mathrm{Tr}(I) = n, \gamma_\mu\gamma^\mu = nI, g^\mu_\mu = n$, etc..

– The ultraviolet divergences appear as poles in $1/(n-4) = 1/\epsilon$ when $\epsilon \to 0$.

In this case, one has would have for the integral of the component $\propto g_{\mu\nu}$ at $p^2 = 0$:

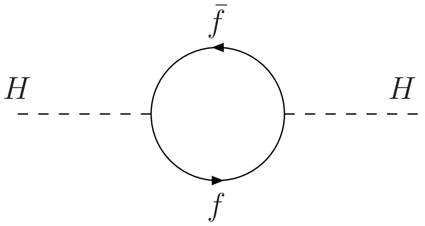
$$\int \frac{d^nk}{(2\pi)^n}\frac{-\frac{1}{2}k^2+m^2}{(k^2-m^2)^2} = -\frac{1}{2}\left[\int \frac{d^nk}{(2\pi)^n}\frac{1}{k^2-m^2} - m^2\int \frac{d^nk}{(2\pi)^n}\frac{1}{(k^2-m^2)^2}\right] \tag{1.35}$$

Since the ultraviolet divergences are poles in $1/\epsilon$, one has for the first integral, $Am^2/\epsilon + \cdots$ for dimensional reasons and the second integral is simply $m^2\partial A/\partial m^2 \sim Am^2/\epsilon$. Therefore the sum of the two integrals gives zero. In fact, with this regularization, the whole self–energy vanishes: $\delta m_\gamma = 0$ at all orders and the photon remains massless.

Hence, this is again an example of a situation where a symmetry has protected a particle [the photon in this case] to acquire a mass through radiative corrections.

1.2.4 The Higgs boson self–energy in the SM

Let us now come to the self–energy of the Higgs boson and consider first the contribution of a heavy fermion f with a number N_f and a coupling $\lambda_f = \sqrt{2} m_f/v$. The diagram below



can be evaluated in the usual way:

$$-i\Sigma_H(p^2) \;\; = \;\; N_f \int \frac{d^4k}{(2\pi)^4} (-1) \mathrm{Tr} \, (\frac{i\lambda_f}{\sqrt{2}}) \frac{i}{\not{k} - m} (\frac{i\lambda_f}{\sqrt{2}}) \frac{i}{\not{p} + \not{k} - m} \tag{1.36}$$

Performing the usual calculation, with the simplifying assumption that the fermion is very heavy compared to the Higgs boson mass which allows to set the external Higgs momentum squared to be zero, $p^2 = M_H^2 = 0$, and using a cut–off Λ for the integral on k^2, one obtains:

$$\Sigma_H(p^2 = 0) \;\; = \;\; 4N_f \left(\frac{\lambda_f}{\sqrt{2}} \right)^2 \frac{1}{16\pi^2} \int_0^1 dx \int_0^{\Lambda^2} dy \, \frac{y(-y + m_f^2)}{(y + m_f^2)^2} \tag{1.37}$$

After the trivial integral on the variable x and the one on y, one gets

$$\Delta M_H^2 = \;\; = \;\; N_f \frac{\lambda_f^2}{8\pi^2} \Big[-\Lambda^2 + 6m_f^2 \mathrm{Log} \frac{\Lambda}{m_f} - 2m_f^2 \Big] + \mathcal{O}(1/\Lambda^2) \tag{1.38}$$

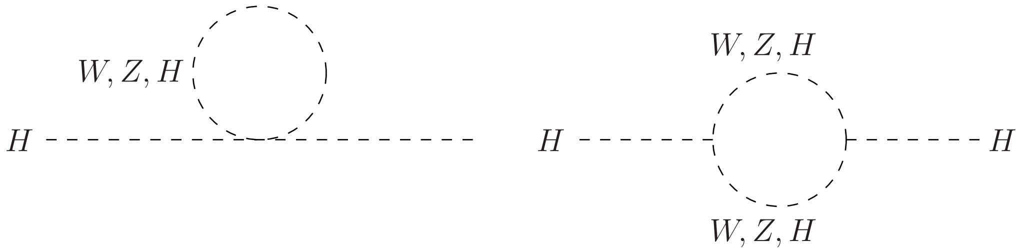
We have thus a quadratically divergent contribution to the Higgs boson mass squared, $\delta M_H^2 \sim \Lambda^2$. This divergence is independent of M_H, and does not disappear if M_H=0. This has to be expected since the choice M_H=0 does not increase the symmetry of the Lagrangian. Contrary to the case of the photon self–energy, the introduction of the cut–off Λ does not break any symmetry and the problem is not solved with dimensional regularization [though we will have only poles in $1/\epsilon$, and the quadratic divergence is not apparent].

Now if we chose the cut–off scale Λ to be $M_{\rm GUT}$ or M_P, the Higgs boson mass which is supposed to lie in the range of electroweak symmetry breaking, $v \sim 250$ GeV, will be huge, $M_H \sim 10^{14}$ to 10^{16} GeV: the Higgs boson mass prefers to be close to the very high scale. This is what is called <u>the hierarchy problem</u>.

Since we want the Higgs boson to be relatively light ($\lesssim$ 1 TeV) for unitarity reasons for instance, we need thus to have $M_H^2|^{\rm physical} = M_H^2|^0 + \Delta M_H^2 +$ counterterm, and adjust this counterterm with a precision of 10^{30}. This is what is called <u>the naturalness problem</u>.

In a complete theory, there is formally no problem: one can adjust the bare value of M_H and the counterterm which are both infinite, to have the physical mass which is finite; this is the case of the logarithmic divergence of m_e in QED for instance. However, we would like to give a physical meaning to the cut–off Λ. In addition, logarithmic and quadratic divergences are of different nature.

In the Standard Model of the electroweak interactions, besides the fermions, there are also contributions to the Higgs boson mass M_H from the massive gauge bosons and from the Higgs boson itself. The Feynman diagrams are:



The total contributions of fermions and bosons in the SM at one–loop order is

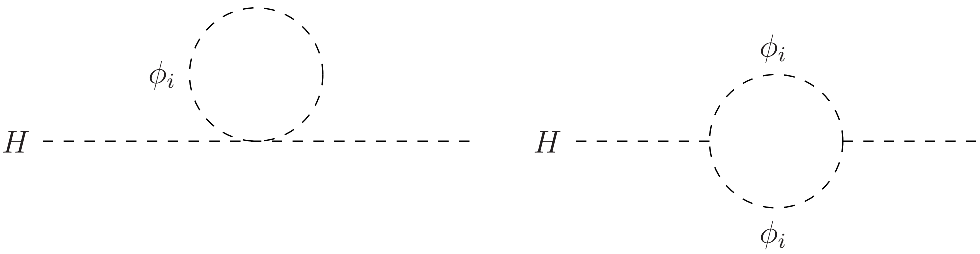
$$\Delta M_H^2 \propto \frac{\alpha}{\pi}\left[3(M_W^2 + M_Z^2 + M_H^2)/4 - \sum m_f^2\right](\Lambda^2/M_W^2) \tag{1.39}$$

One can adjust the unknown value of M_H so that the quadratic divergence when summed over fermions and bosons disappears and this would be a prediction for the Higgs mass; numerically, one obtains $M_H \sim 300$ GeV for a top quark mass of $m_t = 175$ GeV and $M_W \simeq 80$ GeV, $M_Z \simeq 91.2$ GeV. However, this will not work at the two–loop level or at higher orders: the quadratic divergence has to be canceled order by order in perturbation theory and the contributions at each order are not the same and do not give a solution or a prediction for M_H.

In summary, the problem of the quadratic divergences to the Higgs boson mass remains in the SM. There is no symmetry which protects M_H in the SM.

1.2.5 Scalar contributions to the Higgs boson self–energy

Let us now assume the existence of a number N_S of two scalar particles with masses m_1 and m_2 and with trilinear and quadrilinear couplings to the Higgs bosons given in subsection 1.1. They contribute to the self–energy of the Higgs boson via the diagrams shown below.



The contribution of these two diagrams to the Higgs boson mass is given by

$$\begin{aligned}\Delta M_H^2 = \Sigma_H(p^2) &= (i)N_S \int \frac{d^4k}{(2\pi)^4}(i\lambda_S)\left[\frac{i}{k^2-m_1^2}+\frac{i}{k^2-m_2^2}\right] \\ &\quad + N_S(i\lambda_S v)^2 \int \frac{d^4k}{(2\pi)^4}\left[\frac{i}{k^2-m_1^2}\frac{i}{(k+p)^2-m_1^2}+m_1 \leftrightarrow m_2\right] \quad (1.40)\end{aligned}$$

Here, because we are in a scalar case [no Dirac algebra], the calculation is really simple. One obtains after the integration

$$\begin{aligned}\Delta M_H^2 &= \frac{\lambda_S N_S}{16\pi^2}\left[-2\Lambda^2+2m_1^2\log\left(\frac{\Lambda}{m_1}\right)+2m_2^2\log\left(\frac{\Lambda}{m_2}\right)\right] \\ &- \frac{\lambda_S^2 v^2 N_S}{16\pi^2}\left[-2+2\log\left(\frac{\Lambda}{m_1}\right)+2\log\left(\frac{\Lambda}{m_2}\right)\right]+\mathcal{O}\left(\frac{1}{\Lambda^2}\right)\end{aligned}$$

Here again, the quadratic divergences are present. But let us now make the following assumptions:

- The Higgs couplings of the scalar particles are related to the Higgs–fermion couplings such that $\lambda_f^2 = -\lambda_S$ (the minus sign is very important!).
- The multiplicative factors for fermions and scalars are the same: $N_S = N_f$ (recall that we have two scalar particles for one fermion).
- To simplify the discussion, the scalars have the same mass: $m_1 = m_2 = m_S$.

Adding the fermionic contribution discussed previously, and the scalar contributions above, and taking into account the previous assumptions, one obtains for the total radiative correction to the Higgs boson mass:

$$\Delta M_H^2|_{\rm tot} = \frac{\lambda_f^2 N_f}{4\pi^2}\left[(m_f^2-m_S^2)\log\left(\frac{\Lambda}{m_S}\right)+3m_f^2\log\left(\frac{m_S}{m_f}\right)\right]$$

As can be seen, the separate quadratic divergences of fermions and scalars have disappeared in the sum. The logarithmic divergence is still present, but even for values $\Lambda = M_P$, the contribution is rather small. This divergence disappears also if in addition we assume that the fermion and the scalars have the same mass $m_S = m_f$. In fact, in this case, the total correction vanishes.

In conclusion, if there are scalar particles with a symmetry which relates their couplings to the couplings of the standard fermions, there is no quadratic divergence to the Higgs boson mass: the hierarchy and naturalness problems are technically solved. If, in addition, there is a "Supersymmetry", which makes that the scalar particle masses are equal to the fermion mass, there are no divergences at all since even the logarithmic ones disappear. The Higgs boson mass is thus protected by this Supersymmetry.

1.3 Supersymmetry and the MSSM

Supersymmetry (SUSY), which predicts the existence of a partner of different spin to every known particle, is the most attractive extension of the SM. It not only stabilizes the huge hierarchy between the GUT and electroweak scales by canceling the radiative corrections to the Higgs boson masses, but also it allows to understand the origin of the hierarchy in terms of radiative gauge symmetry breaking. Moreover, SUSY models allow for a consistent unification of the three gauge couplings and offer a natural solution to the Dark Matter problem. In this subsection, we recall very briefly the basic features of (SUSY) and of the Minimal Supersymmetric extension of the Standard Model (MSSM). For a more detailed discussion see Ref. [3].

1.3.1 Basics of Supersymmetry

Supersymmetry (SUSY) is a symmetry relating particles of integer spin, i.e. bosons, and particles of spin 1/2, i.e. fermions. The SUSY generators $\mathcal{Q}$ transform fermions into bosons and vice–versa:

$$\mathcal{Q}|\text{Fermion}>=|\text{Boson}> \quad , \quad \mathcal{Q}|\text{Boson}>=|\text{Fermion}> \tag{1.41}$$

When the symmetry is exact, the bosonic fields (the scalar and gauge fields of respectively, spin 0 and spin 1) and the fermionic fields (spin 1/2) have the same masses and same quantum numbers, except for the spin. The particles are combined into superfields. The simplest case is the chiral or scalar superfield which contains a complex scalar field S with two degrees of freedom and a Majorana fermionic field with two components ζ ($\zeta^c = \zeta$). Another possibility is the vector superfield, containing a massless gauge field A_μ^a with a the gauge index and a Majorana fermionic field with 2 components λ_a.

All fields involved have the canonical kinetic energies given by the Lagrangian

$$\mathcal{L}_{\text{kin}} = \Sigma_i \Big\{ (D_\mu S_i^*)(D^\mu S_i) + i\overline{\psi}_i D_\mu \gamma^\mu \psi_i \Big\} + \Sigma_a \Big\{ -\frac{1}{4} F_{\mu\nu}^a F^{\mu\nu a} + \frac{i}{2} \overline{\lambda}_a D \lambda_a \Big\} \tag{1.42}$$

with D the usual covariant derivative [note that the fields $\psi(\lambda)$ are in 4(2) dimensions]. The interactions among the fields are specified by SUSY and gauge invariance

$$\begin{aligned} \mathcal{L}_{\text{int. scal-fer.-gauginos}} &= -\sqrt{2} \sum_{i,a} g_a \Big[S_i^* T^a \overline{\psi}_{iL} \lambda_a + \text{h.c.} \Big] \\ \mathcal{L}_{\text{int. quartic scal.}} &= -\frac{1}{2} \sum_a \Big(\sum_i g_a S_i^* T^a S_i \Big)^2 \end{aligned} \tag{1.43}$$

and as can be seen, all interactions are given in terms of the gauge coupling constants. Thus, when SUSY is exact, everything is completely specified and there is no adjustable parameter.

The only freedom that one has is the choice of the Superpotential W which gives the form of the scalar potential and the Yukawa interactions between fermion and scalar fields. However, the Superpotential should be invariant under SUSY and gauge transformations. It should obey the following three conditions:

– It must be a function of the superfields z_i only and not their conjugate z_i^*.

– It should be an analytic function and therefore, it has no derivative interaction.

– It should have only terms of dimension 2 and 3 to keep the theory renormalizable.

In terms of the Superpotential W, the interaction Lagrangian may be written as

$$\Rightarrow \mathcal{L}_W = -\sum_i \mid \frac{\partial W}{\partial z_i} \mid^2 - \frac{1}{2} \sum_{ij} \Big[\overline{\psi}_{iL} \frac{\partial^2 W}{\partial z_i \partial z_j} \psi_j + \text{h.c.} \Big] \tag{1.44}$$

A simple way to obtain the interactions explicitly is to take the derivative of W with respect to the fields z_i, and then evaluate in terms of the scalar fields S_i.

However, SUSY cannot be an exact symmetry since there are no fundamental scalar particles which have the same mass as the known fermions [in fact, no fundamental scalar particle has been observed yet!]. Therefore, SUSY must be explicitly broken. In addition, we need the SUSY breaking to occur at rather low energies to solve the three problems which we have with the Standard Model and which are:

– The hierarchy problem related to the quadratic divergences in the Higgs sector.

– The unification of the three coupling constants of $\mathrm{SU(3)_C \times SU(2)_L \times U(1)_Y}$.

– The dark matter problem which calls for the existence of a massive stable particle.

However, in the breaking, we still need to preserve the gauge invariance and the renormalizability of the theory, and the fact that there are still no quadratic divergences to the Higgs boson masses. Since up to now, there is no satisfactory dynamical way to break SUSY, a possibility is to break the Supersymmetry by hand (which leads to several options). This gives a low energy effective SUSY theory, the most economic version being the Minimal Supersymmetric Standard Model (MSSM) that we will discuss below.

1.3.2 The unconstrained Minimal Supersymmetric Standard Model

The unconstrained MSSM is defined usually by the following four basic assumptions:

(a) Minimal gauge group:

The MSSM is based on the group $\mathrm{SU(3)_C \times SU(2)_L \times U(1)_Y}$, i.e. the SM symmetry. SUSY implies then that the spin–1 gauge bosons and their spin–1/2 partners, the gauginos [bino $\tilde{B}$, winos $\tilde{W}_{1-3}$ and gluinos $\tilde{G}_{1-8}$], are in vector supermultiplets; see the table below.

Superfields	$SU(3)_C$	$SU(2)_L$	$U(1)_Y$	Particle content
$\hat{G}^a$	8	1	0	G^μ, $\tilde{g}$
$\hat{W}^i$	1	3	0	W_i^μ, $\tilde{\omega}_i$
$\hat{B}$	1	1	0	B^μ, $\tilde{b}$

(b) Minimal particle content:

There are only three generations of spin–1/2 quarks and leptons [no right–handed neutrino] as in the SM. The left– and right–handed chiral fields belong to chiral superfields together with their spin–0 SUSY partners, the squarks and sleptons: $\hat{Q}, \hat{u}_R, \hat{d}_R, \hat{L}, \hat{l}_R$. In addition, two chiral superfields $\hat{H}_d$, $\hat{H}_u$ with respective hypercharges -1 and $+1$ are needed for the cancellation of chiral anomalies. Their scalar components, H_2 and H_1, give separately masses to the isospin $+1/2$ and $-1/2$ fermions in a SUSY invariant way [recall that the SUSY potential does should involve conjugate fields and we cannot generate with the same doublet the masses of both types of fermions]. The various fields are summarized in the table below. As will be discussed later, the two doublets fields lead to five Higgs particles: two CP–even h, H bosons, a pseudoscalar A boson and two charged $H^\pm$ bosons. Their spin–1/2 superpartners, the higgsinos, will mix with the winos and the bino, to give the "ino" mass eigenstates: the two charginos $\chi_{1,2}^\pm$ and the four neutralinos $\chi_{1,2,3,4}^0$.

Superfield	$SU(3)_C$	$SU(2)_L$	$U(1)_Y$	Particle content
$\hat{Q}$	3	2	$\frac{1}{3}$	(u_L, d_L), $(\tilde{u}_L, \tilde{d}_L)$
$\hat{U}^c$	$\overline{3}$	1	$-\frac{4}{3}$	$\overline{u}_R$, $\tilde{u}_R^*$
$\hat{D}^c$	$\overline{3}$	1	$\frac{2}{3}$	$\overline{d}_R$, $\tilde{d}_R^*$
$\hat{L}$	1	2	-1	(ν_L, e_L), $(\tilde{\nu}_L, \tilde{e}_L)$
$\hat{E}^c$	1	1	2	$\overline{e}_R$, $\tilde{e}_R^*$
$\hat{H}_1$	1	2	-1	$(H_1, \tilde{h}_1)$
$\hat{H}_2$	1	2	1	$(H_2, \tilde{h}_2)$

(c) Minimal Yukawa interactions and R–parity conservation:

To enforce lepton and baryon number conservation, a discrete and multiplicative symmetry called R–parity is imposed. It is defined by $R_p = (-1)^{2s+3B+L}$, where L and B are the lepton and baryon numbers and s is the spin quantum number. The R–parity quantum numbers are then $R_p = +1$ for the ordinary particles [fermions, gauge and Higgs bosons], and $R_p = -1$ for their supersymmetric partners. In practice, the conservation of R–parity has important consequences:

– the SUSY particles are always produced in pairs,
– in their decay products there is always an odd number of SUSY particles,
– the lightest SUSY particle (LSP) is absolutely stable.

The three conditions listed above are sufficient to completely determine a globally supersymmetric Lagrangian. The kinetic part of the Lagrangian is obtained by generalizing the notion of covariant derivative to the SUSY case. The most general superpotential, compatible with gauge invariance, renormalizability and R–parity conservation is written as:

$$W = \sum_{i,j=gen} -Y^u_{ij}\, \hat{u}_{Ri} \hat{H}_2.\hat{Q}_j + Y^d_{ij}\, \hat{d}_{Ri} \hat{H}_1.\hat{Q}_j + Y^l_{ij}\, \hat{l}_{Ri} \hat{H}_1.\hat{L}_j + \mu \hat{H}_1.\hat{H}_2 \tag{1.45}$$

The product between $\mathrm{SU(2)_L}$ doublets reads $H.Q \equiv \epsilon_{ab} H^a Q^b$ where a, b are $\mathrm{SU(2)_L}$ indices and $\epsilon_{12} = 1 = -\epsilon_{21}$, and $Y^{u,d,l}_{ij}$ denote the Yukawa couplings among generations. The first three terms in the previous expression are nothing else but a superspace generalization of the Yukawa interaction in the SM, while the last term is a globally supersymmetric Higgs mass term. The supersymmetric part of the tree–level potential $V_{\rm tree}$ is the sum of the so–called F– and D–terms, where the F–terms come from the superpotential through derivatives with respect to all scalar fields ϕ_a,

$$V_F = \sum_a |W^a|^2 \ \text{ with } W^a = \partial W / \partial \phi_a \tag{1.46}$$

and the D–terms corresponding to the $\mathrm{U(1)_Y}, \mathrm{SU(2)_L}$ and $\mathrm{SU(3)_C}$ symmetries are given by

$$V_D = \frac{1}{2} \sum_{i=1}^{3} \left(\sum_a g_i \phi_a^* T^i \phi_a \right)^2 \tag{1.47}$$

with T^i and g_i being the generators and coupling constants of the corresponding groups.

(d) Minimal set of soft SUSY–breaking terms:

Finally, to break Supersymmetry, while preventing the reappearance of the quadratic divergences [this is what is soft breaking], one adds to the supersymmetric Lagrangian a set of terms which explicitly but softly break SUSY:

- Mass terms for the gluinos, winos and binos:

$$-\mathcal{L}_{\rm gaugino} = \frac{1}{2} \left[M_1 \tilde{B} \tilde{B} + M_2 \sum_{a=1}^{3} \tilde{W}^a \tilde{W}_a + M_3 \sum_{a=1}^{8} \tilde{G}^a \tilde{G}_a \ + \ \text{h.c.} \right] \tag{1.48}$$

- Mass terms for the scalar fermions:

$$-\mathcal{L}_{\rm sfermions} = \sum_{i=gen} m^2_{\tilde{Q}i} \tilde{Q}_i^\dagger \tilde{Q}_i + m^2_{\tilde{L}i} \tilde{L}_i^\dagger \tilde{L}_i + m^2_{\tilde{u}i} |\tilde{u}_{R_i}|^2 + m^2_{\tilde{d}i} |\tilde{d}_{R_i}|^2 + m^2_{\tilde{l}i} |\tilde{l}_{R_i}|^2 \tag{1.49}$$

- Mass and bilinear terms for the Higgs bosons:

$$-\mathcal{L}_{\rm Higgs} = m^2_{H_2} H_2^\dagger H_2 + m^2_{H_1} H_1^\dagger H_1 + B\mu (H_2.H_1 + \text{h.c.}) \tag{1.50}$$

- Trilinear couplings between sfermions and Higgs bosons

$$-\mathcal{L}_{\text{tril.}} = \sum_{i,j=gen} \left[A^u_{ij} Y^u_{ij} \tilde{u}_{R_i} H_2.\tilde{Q}_j + A^d_{ij} Y^d_{ij} \tilde{d}_{R_i} H_1.\tilde{Q}_j + A^l_{ij} Y^l_{ij} \tilde{l}_{R_i} H_1.\tilde{L}_j \ + \ \text{h.c.} \right] \quad (1.51)$$

The soft SUSY–breaking scalar potential is the sum of the three last terms:

$$V_{\text{soft}} = -\mathcal{L}_{\text{sfermions}} - \mathcal{L}_{\text{Higgs}} - \mathcal{L}_{\text{tril.}} \quad (1.52)$$

Up to now, no constraint is applied to this Lagrangian, although for generic values of the parameters, it might lead to severe phenomenological problems, such as flavor changing neutral currents [FCNC] and unacceptable amount of additional CP–violation color and charge breaking minima an incorrect value of the Z boson mass, etc... The MSSM defined by the four hypotheses (a)–(d) above, is generally called the unconstrained MSSM.

In the unconstrained MSSM, and in the general case where one allows for intergenerational mixing and complex phases, the soft SUSY breaking terms will introduce a huge number (105) of unknown parameters, in addition to the 19 parameters of the SM. This large number of free parameters makes any phenomenological analysis in the general MSSM very complicated. In addition, many "generic" sets of these parameters are excluded by the severe phenomenological constraints discussed above. A phenomenologically viable MSSM can be defined by making the following three assumptions: (i) All the soft SUSY–breaking parameters are real and therefore there is no new source of CP–violation generated, in addition to the one from the CKM matrix. (ii) The matrices for the sfermion masses and for the trilinear couplings are all diagonal, implying the absence of FCNCs at the tree–level. (iii) First and second sfermion generation universality at low energy to cope with the severe constraints from K^0–$\bar{K}^0$ mixing, etc.

Making these three assumptions will lead to 22 input parameters only:

$\tan\beta$: the ratio of the vevs of the two–Higgs doublet fields.
$m^2_{H_1}, m^2_{H_2}$: the Higgs mass parameters squared.
M_1, M_2, M_3: the bino, wino and gluino mass parameters.
$m_{\tilde{q}}, m_{\tilde{u}_R}, m_{\tilde{d}_R}, m_{\tilde{l}}, m_{\tilde{e}_R}$: the first/second generation sfermion mass parameters.
$m_{\tilde{Q}}, m_{\tilde{t}_R}, m_{\tilde{b}_R}, m_{\tilde{L}}, m_{\tilde{\tau}_R}$: the third generation sfermion mass parameters.
A_u, A_d, A_e: the first/second generation trilinear couplings.
A_t, A_b, A_τ: the third generation trilinear couplings.

Two remarks can be made at this stage: (i) The Higgs–higgsino (supersymmetric) mass parameter $|\mu|$ (up to a sign) and the soft SUSY–breaking bilinear Higgs term B are determined, given the above parameters, through the electroweak symmetry breaking conditions as will be discussed later. Alternatively, one can trade the values of $m^2_{H_1}$ and $m^2_{H_2}$ with the "more physical" pseudoscalar Higgs boson mass M_A and parameter μ. (*ii*) Since the trilinear sfermion couplings will be always multiplied by the fermion masses, they are in general important only in the case of the third generation.

Such a model, with this relatively moderate number of parameters [especially that, in general, only a small subset appears when one looks at a given sector of the model] has much more predictability and is much easier to investigate phenomenologically, compared to the unconstrained MSSM. One can refer to this 22 free input parameters model as the "phenomenological" MSSM or pMSSM.

1.3.3 mSUGRA and the constrained MSSMs

Almost all problems of the general or unconstrained MSSM are solved at once if the soft SUSY–breaking parameters obey a set of universal boundary conditions at the GUT scale. If one takes these parameters to be real, this solves all potential problems with CP violation as well. The underlying assumption is that SUSY–breaking occurs in a hidden sector which communicates with the visible sector only through gravitational–strength interactions, as specified by Supergravity. Universal soft breaking terms then emerge if these Supergravity interactions are "flavor–blind" [like ordinary gravitational interactions]. This is assumed to be the case in the constrained MSSM or minimal Supergravity (mSUGRA) model.

Besides the unification of the gauge coupling constants $g_{1,2,3}$ of the U(1), SU(2) and SU(3) groups, which is verified given the experimental results from LEP1 and which can be viewed as fixing the Grand Unification scale $M_{\rm GUT} \sim 2 \cdot 10^{16}$ GeV, the unification conditions in mSUGRA, are as follows:

– Unification of the gaugino [bino, wino and gluino] masses:

$$M_1(M_{\rm GUT}) = M_2(M_{\rm GUT}) = M_3(M_{\rm GUT}) \equiv m_{1/2} \tag{1.53}$$

– Universal scalar [i.e. sfermion and Higgs boson] masses [i is the generation index]:

$$\begin{aligned} M_{\tilde{Q}_i}(M_{\rm GUT}) &= M_{\tilde{u}_{Ri}}(M_{\rm GUT}) = M_{\tilde{d}_{Ri}}(M_{\rm GUT}) = M_{\tilde{L}_i}(M_{\rm GUT}) = M_{\tilde{l}_{Ri}}(M_{\rm GUT}) \\ &= M_{H_u}(M_{\rm GUT}) = M_{H_d}(M_{\rm GUT}) \equiv m_0 \end{aligned} \tag{1.54}$$

– Universal trilinear couplings:

$$A^u_{ij}(M_{\rm GUT}) = A^d_{ij}(M_{\rm GUT}) = A^l_{ij}(M_{\rm GUT}) \equiv A_0\, \delta_{ij} \tag{1.55}$$

Besides the three parameters $m_{1/2}, m_0$ and A_0, the supersymmetric sector is described at the GUT scale by the bilinear coupling B and the supersymmetric Higgs(ino) mass parameter μ. However, one has to require that EWSB takes place at some low energy scale. This results in two necessary minimization conditions of the two–Higgs doublet scalar potential which fix the values μ^2 and $B\mu$. The sign of μ is not determined. Therefore, in this model, one is left with only four continuous free parameters, and an unknown sign:

$$\tan\beta\ ,\ m_{1/2}\ ,\ m_0\ ,\ A_0\ ,\ \ \mathrm{sign}(\mu). \tag{1.56}$$

All the soft SUSY breaking parameters at the weak scale are then obtained through Renormalization Group Equations (RGEs).

There also other constrained MSSM scenarios and we will briefly mention two of them:

Anomaly Mediated Supersymmetry Breaking (AMSB) models:

In AMSB models, the SUSY–breaking occurs also in a hidden sector, but it is transmitted to the visible sector by the super–Weyl anomaly. The gaugino, scalar masses and trilinear couplings are then simply related to the scale dependence of the gauge and matter kinetic functions. This leads to soft SUSY–breaking scalar masses for the first two generation sfermions that are almost diagonal [when the small Yukawa couplings are neglected] which solves the SUSY flavor problem which affects mSUGRA for instance. In these models, the soft SUSY breaking terms are given in terms of the gravitino mass $m_{3/2}$, the β functions for the gauge and Yukawa couplings g_a and Y_i and the anomalous dimensions γ_i of the chiral superfields. One then has, in principle, only three input parameters $m_{3/2}, \tan\beta$ and sign(μ) [μ^2 and B are obtained as usual by requiring correct EWSB]. However, this picture is spoiled by the fact that the anomaly mediated contribution to the slepton scalar masses squared is negative. This problem can be cured by adding a positive non–anomaly mediated contribution to the soft masses, an m_0^2 term at $M_{\rm GUT}$, as in mSUGRA.

Gauge Mediated Supersymmetry Breaking (GMSB) models:

In GMSB models, SUSY–breaking is transmitted to the MSSM fields via the SM gauge interactions. In the original scenario, the model consists of three distinct sectors: a secluded sector where SUSY is broken, a "messenger" sector containing a singlet field and messenger fields with $\mathrm{SU(3)_c \times SU(2)_L \times U(1)_Y}$ quantum numbers, and a sector containing the fields of the MSSM. Another possibility, the so–called "direct gauge mediation" has only two sectors: one which is responsible for the SUSY breaking and contains the messenger fields, and another sector consisting of the MSSM fields. In both cases, the soft SUSY–breaking masses for the gauginos and squared masses for the sfermions arise, respectively, from one–loop and two–loop diagrams involving the exchange of the messenger fields, while the trilinear Higgs–sfermion–sfermion couplings can be taken to be negligibly small at the messenger scale since they are [and not their square as for the sfermion masses] generated by two–loop gauge interactions. This allows an automatic and natural suppression of FCNC and CP–violation.

1.3.4 The Supersymmetric particle spectrum

Let us now discuss the general features of the chargino/neutralino and sfermion sectors of the MSSM. The Higgs sector of the MSSM will be discussed in much more details later.

a) The chargino/neutralino sector:

The general chargino mass matrix depends on the parameters M_2, μ and $\tan\beta$

$$\mathcal{M}_C = \left[\begin{array}{cc} M_2 & \sqrt{2} M_W \sin\beta \\ \sqrt{2} M_W \cos\beta & \mu \end{array} \right] \tag{1.57}$$

is diagonalized by two real matrices U and V,

$$U^* \mathcal{M}_C V^{-1} \quad \rightarrow \quad U = \mathcal{O}_- \text{ and } V = \left\{ \begin{array}{ll} \mathcal{O}_+ & \text{if } \det\mathcal{M}_C \geq 0 \\ \sigma_3 \mathcal{O}_+ & \text{if } \det\mathcal{M}_C < 0 \end{array} \right. \tag{1.58}$$

where the σ_3 matrix and the $\mathcal{O}_\pm$ matrices are given by [with the appropriate signs depending on the values of M_2, μ, and $\tan\beta$]

$$\sigma_3 = \left[\begin{array}{cc} +1 & 0 \\ 0 & -1 \end{array} \right] \quad , \quad \mathcal{O}_\pm = \left[\begin{array}{cc} \cos\theta_\pm & \sin\theta_\pm \\ -\sin\theta_\pm & \cos\theta_\pm \end{array} \right] \tag{1.59}$$

with

$$\begin{aligned} \tan 2\theta_- &= \frac{2\sqrt{2} M_W (M_2 \cos\beta + \mu \sin\beta)}{M_2^2 - \mu^2 - 2M_W^2 \cos\beta} \\ \tan 2\theta_+ &= \frac{2\sqrt{2} M_W (M_2 \sin\beta + \mu \cos\beta)}{M_2^2 - \mu^2 + 2M_W^2 \cos\beta} \end{aligned} \tag{1.60}$$

This leads to the two chargino masses,

$$\begin{aligned} m_{\chi^+_{1,2}} = \quad & \frac{1}{\sqrt{2}} \left[M_2^2 + \mu^2 + 2M_W^2 \right. \\ & \left. \mp \left\{ (M_2^2 - \mu^2)^2 + 4M_W^4 \cos^2 2\beta + 4M_W^2 (M_2^2 + \mu^2 + 2M_2 \mu \sin 2\beta) \right\}^{\frac{1}{2}} \right]^{\frac{1}{2}} \end{aligned} \tag{1.61}$$

In the limit $|\mu| \gg M_2, M_Z$, the masses of the two charginos reduce to [$\epsilon_\mu \equiv \text{sign}(\mu)$]

$$\begin{aligned} m_{\chi_1^+} &\simeq M_2 - \frac{M_W^2}{\mu^2} (M_2 + \mu \sin 2\beta) \\ m_{\chi_2^+} &\simeq |\mu| + \frac{M_W^2}{\mu^2} \epsilon_\mu (M_2 \sin 2\beta + \mu) \end{aligned} \tag{1.62}$$

For $|\mu| \rightarrow \infty$, the lightest chargino corresponds to a pure wino state with $m_{\chi_1^+} \simeq M_2$, while the heavier chargino corresponds to a pure higgsino state with $m_{\chi_2^+} = |\mu|$.

In the case of the neutralinos, the four–dimensional neutralino mass matrix depends on the same two mass parameters μ and M_2, if the GUT relation $M_1 = \frac{5}{3}\tan^2\theta_W \, M_2 \simeq \frac{1}{2}M_2$ is used. In the $(-i\tilde{B}, -i\tilde{W}_3, \tilde{H}_1^0, \tilde{H}_2^0)$ basis, it has the form $[s_W^2 = 1 - c_W^2 \equiv \sin^2\theta_W]$

$$\mathcal{M}_N = \begin{bmatrix} M_1 & 0 & -M_Z s_W \cos\beta & M_Z s_W \sin\beta \\ 0 & M_2 & M_Z c_W \cos\beta & -M_Z c_W \sin\beta \\ -M_Z s_W \cos\beta & M_Z c_W \cos\beta & 0 & -\mu \\ M_Z s_W \sin\beta & -M_Z c_W \sin\beta & -\mu & 0 \end{bmatrix} \tag{1.63}$$

It can be diagonalized analytically by a single real matrix Z; the [positive] masses of the neutralino states $m_{\chi_i^0}$ have complicated expressions which will not be given here. In the limit of large $|\mu|$ values, the masses of the neutralino states however simplify to

$$\begin{aligned} m_{\chi_1^0} &\simeq M_1 - \frac{M_Z^2}{\mu^2}\left(M_1 + \mu\sin 2\beta\right) s_W^2 \\ m_{\chi_2^0} &\simeq M_2 - \frac{M_Z^2}{\mu^2}\left(M_2 + \mu\sin 2\beta\right) c_W^2 \\ m_{\chi_3^0} &\simeq |\mu| + \frac{1}{2}\frac{M_Z^2}{\mu^2}\epsilon_\mu(1-\sin 2\beta)\left(\mu + M_2 s_W^2 + M_1 c_W^2\right) \\ m_{\chi_4^0} &\simeq |\mu| + \frac{1}{2}\frac{M_Z^2}{\mu^2}\epsilon_\mu(1+\sin 2\beta)\left(\mu - M_2 s_W^2 - M_1 c_W^2\right) \end{aligned} \tag{1.64}$$

Again, for $|\mu| \to \infty$, two neutralinos are pure gaugino states with masses $m_{\chi_1^0} \simeq M_1$, $m_{\chi_2^0} = M_2$, while the two others are pure higgsino states, with masses $m_{\chi_3^0} \simeq m_{\chi_4^0} \simeq |\mu|$.

The evolution of the gaugino masses are related to those of the gauge couplings constant and, choosing a common value $m_{1/2}$ at the GUT scale, one obtains at the weak scale:

$$M_3 : M_2 : M_1 \sim 6 : 2 : 1 \tag{1.65}$$

The gluino mass is identified with M_3 at the tree–level.

b) The sfermion sector:

Assuming a universal scalar mass m_0 and gaugino mass $m_{1/2}$ at the GUT scale, one obtains relatively simple expressions for the left– and right–handed sfermion masses when performing the RGE evolution to the weak scale at one–loop order, if the the Yukawa couplings in the RGE's are neglected [for third generation squarks this is a poor approximation since these couplings can be large; in this case numerical analyzes are]. One has:

$$m_{\tilde{f}_{L,R}}^2 = m_0^2 + \sum_{i=1}^{3} F_i(f) m_{1/2}^2 \pm (I_f^3 - e_f s_W^2) M_Z^2 \cos 2\beta \tag{1.66}$$

I_f^3 and e_f are the weak isospin and the electric charge of the sfermion and F_i are the RGE coefficients for the three gauge couplings at the scale $Q \sim M_Z$, given by

$$F_i = \frac{c_i(f)}{b_i}\left[1 - \left(1 - \frac{\alpha_U}{4\pi} b_i \log\frac{Q^2}{M_U^2}\right)^{-2}\right] \tag{1.67}$$

The coefficients b_i, assuming that all the MSSM particle spectrum contributes to the evolution from Q to the GUT scale M_G, are given by: $b_1 = 33/5, b_2 = 1, b_3 = -3$. The coefficients $c(\tilde{f}) = (c_1, c_2, c_3)(\tilde{f})$ depend on the hypercharge and color of the sfermions

$$c(\tilde{L}) = \begin{pmatrix} \frac{3}{10} \\ \frac{3}{2} \\ 0 \end{pmatrix},\ c(\tilde{l}_R) = \begin{pmatrix} \frac{6}{5} \\ 0 \\ 0 \end{pmatrix},\ c(\tilde{Q}) = \begin{pmatrix} \frac{1}{30} \\ \frac{3}{2} \\ \frac{8}{3} \end{pmatrix},\ c(\tilde{u}_R) = \begin{pmatrix} \frac{8}{15} \\ 0 \\ \frac{8}{3} \end{pmatrix},\ c(\tilde{d}_R) = \begin{pmatrix} \frac{2}{15} \\ 0 \\ \frac{8}{3} \end{pmatrix}$$

With the input gauge coupling constants at the scale of the Z boson mass $\alpha_1(M_Z) \simeq 0.01, \alpha_2(M_Z) \simeq 0.033$ and $\alpha_3(M_Z) \simeq 0.118$, one obtains $M_U \sim 1.9 \times 10^{16}$ GeV for the GUT scale and $\alpha_U = 0.041$ for the coupling constant α_U. Using these values, one obtains for the left– and right–handed sfermions

$$\begin{aligned}
m^2_{\tilde{u}_L} &= m_0^2 + 6.28 m_{1/2}^2 + 0.35 M_Z^2 \cos 2\beta \\
m^2_{\tilde{d}_L} &= m_0^2 + 6.28 m_{1/2}^2 - 0.42 M_Z^2 \cos 2\beta \\
m^2_{\tilde{u}_R} &= m_0^2 + 5.87 m_{1/2}^2 + 0.16 M_Z^2 \cos 2\beta \\
m^2_{\tilde{d}_R} &= m_0^2 + 5.82 m_{1/2}^2 - 0.08 M_Z^2 \cos 2\beta \\
m^2_{\tilde{\nu}_L} &= m_0^2 + 0.52 m_{1/2}^2 + 0.50 M_Z^2 \cos 2\beta \\
m^2_{\tilde{e}_L} &= m_0^2 + 0.52 m_{1/2}^2 - 0.27 M_Z^2 \cos 2\beta \\
m^2_{\tilde{e}_R} &= m_0^2 + 0.15 m_{1/2}^2 - 0.23 M_Z^2 \cos 2\beta
\end{aligned} \tag{1.68}$$

In the case of the third generation sparticles, left– and right–handed sfermions will mix; for a given sfermion $\tilde{f} = \tilde{t}, \tilde{b}$ and $\tilde{\tau}$, the mass matrices which determine the mixing are

$$M^2_{\tilde{f}} = \begin{bmatrix} m^2_{\tilde{f}_L} + m_f^2 & m_f(A_f - \mu r_f) \\ m_f(A_f - \mu r_f) & m^2_{\tilde{f}_R} + m_f^2 \end{bmatrix} \tag{1.69}$$

where the sfermion masses $m_{\tilde{f}_{L,R}}$ are given above, m_f are the masses of the partner fermions and $r_b = r_\tau = 1/r_t = \tan\beta$. These matrices are diagonalized by orthogonal matrices; the mixing angles θ_f and the squark eigenstate masses are given by

$$\sin 2\theta_f = \frac{2 m_f (A_f - \mu r_f)}{m^2_{\tilde{f}_1} - m^2_{\tilde{f}_2}} \quad , \quad \cos 2\theta_f = \frac{m^2_{\tilde{f}_L} - m^2_{\tilde{f}_R}}{m^2_{\tilde{f}_1} - m^2_{\tilde{f}_2}} \tag{1.70}$$

$$m^2_{\tilde{f}_{1,2}} = m_f^2 + \frac{1}{2}\left[m^2_{\tilde{f}_L} + m^2_{\tilde{f}_R} \mp \sqrt{(m^2_{\tilde{f}_L} - m^2_{\tilde{f}_R})^2 + 4 m_f^2 (A_f - \mu r_f)^2} \right] \tag{1.71}$$

Due to the large value of m_t, the mixing is particularly strong in the stop sector. This generates a large splitting between the masses of the two stop eigenstates, possibly leading to a lightest top squark much lighter than the other squarks and even the top quark.

2. The MSSM Higgs bosons

2.1 The Higgs sector of the MSSM

2.1.1 The electroweak symmetry breaking in the SM

Let us first summarize the electroweak symmetry breaking mechanism in the Standard Model, which has been discussed in detail in the previous lectures [1] and which is also reviewed in detail in the "Higgs Hunters Guide" [4]. In the SM, we have:

– three generations of matter fields, where the left-handed fermions are in weak isodoublets, while the right–handed fermions are in weak isosinglets, and where the hypercharge is defined in terms of weak isospin I_3 and the electric charge Q (in term of the proton charge $+e$) is given by $Y = 2Q - 2I_3$

$$L_1 = \begin{pmatrix} \nu_e \\ e^- \end{pmatrix}_L \; , \; R_1 = e_R^- \; , \; Q_1 = \begin{pmatrix} u \\ d \end{pmatrix}_L \; , \; u_{R1} = u_R \; , \; d_{R1} = d_R \;\; etc... \tag{2.1}$$

– four gauge fields corresponding to the generators of $\mathrm{SU(2)_L \times U(1)_Y}$: the field B_μ, corresponding to the generator of the abelian group $\mathrm{U(1)_Y}$, and the three fields $W_\mu^{1,2,3}$ corresponding to the three generators of the non–abelian $\mathrm{SU(2)_L}$ symmetry group, T^a with a=1,2,3; these generators are in fact equivalent to half of the non-commuting 2×2 Pauli matrices:

$$T^a = \frac{\tau^a}{2} \;\; , \;\; \tau_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \; , \; \tau_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \; , \; \tau_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \tag{2.2}$$

with the commutation relation between these generators given by:

$$[T^a, T^b] = i\epsilon^{abc} T_c \quad \text{and} \quad [B, B] = 0 \tag{2.3}$$

where ϵ^{abc} is the antisymmetric tensor. The field strengths are given by:

$$F_{\mu\nu}^a = \partial_\mu W_\nu^a - \partial_\nu W_\mu^a + g_2 \epsilon^{abc} W_\mu^b W_\nu^c \; , \; B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu \tag{2.4}$$

The matter fields ψ are minimally coupled to the gauge fields through the covariant derivative D_μ defined by:

$$D_\mu \psi = \left(\partial_\mu - i g_2 T_a W_\mu^a - i g_1 \frac{Y}{2} B_\mu \right) \psi \tag{2.5}$$

where g_2 and g_1 are, respectively, the coupling constants of $\mathrm{SU(2)_L}$ and $\mathrm{U(1)_Y}$. Because of the non–abelian nature of the isospin group, there are also self–interactions between the

SU(2) gauge fields. The SM Lagrangian, without mass terms for fermions and gauge bosons, [which when put by hand, spoil the gauge symmetry of the theory] is then given by

$$\begin{aligned}\mathcal{L}_{\rm SM} \;=\;& -\frac{1}{4}F_{\mu\nu}^{a}F_{a}^{\mu\nu} - \frac{1}{4}B_{\mu\nu}B^{\mu\nu} \\ & + \overline{L_i}\, iD_\mu\gamma^\mu\, L_i + \overline{e}_{Ri}\, iD_\mu\gamma^\mu\, e_{R_i} \;+ \overline{Q_i}\, iD_\mu\gamma^\mu\, Q_i + \overline{u}_{Ri}\, iD_\mu\gamma^\mu\, u_{R_i} \;+ \overline{d}_{Ri}\, iD_\mu\gamma^\mu\, d_{R_i}\end{aligned} \tag{2.6}$$

This Lagrangian is invariant under local gauge transformations for fermion and gauge fields:

$$\begin{aligned} L(x) \to L'(x) = e^{i\alpha_a(x)T^a + i\beta(x)Y}\, L(x) \quad , \quad R(x) \to R'(x) = e^{i\beta(x)Y}\, R(x) \\ \vec{W}_\mu(x) \to \vec{W}_\mu(x) - \frac{1}{g_2}\partial_\mu\vec{\alpha}(x) - \vec{\alpha}(x)\times\vec{W}_\mu(x)\ ,\ B_\mu(x) \to B_\mu(x) - \frac{1}{g_1}\partial_\mu\beta(x)\end{aligned} \tag{2.7}$$

Using the Higgs mechanism, we need to generate masses for three gauge bosons $W^\pm$ and Z but the photon should remain massless since QED should stay an exact symmetry. Therefore, we need at least 3 degrees of freedom for the scalar fields. The simplest choice is a complex SU(2) doublet of scalar fields ϕ

$$\Phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} \quad \text{with } Y_\phi = +1 \tag{2.8}$$

To the Lagrangian discussed previously, we need to add the invariant terms of the scalar field part:

$$\mathcal{L}_S = (D^\mu\Phi)^\dagger(D_\mu\Phi) - \mu^2\Phi^\dagger\Phi - \lambda(\Phi^\dagger\Phi)^2 \tag{2.9}$$

If the kinetic term μ^2 is positive, the potential $V(\Phi) = \mu^2\Phi^\dagger\Phi - \lambda(\Phi^\dagger\Phi)^2$ is also positive if the self–coupling λ is positive and the minimum of the potential is obtained for $\Phi_0 = 0$. The Lagrangian $\mathcal{L}_S$ is then, simply the Lagrangian of a spin–zero particle of mass μ. For $\mu^2 < 0$, the doublet field Φ will develop a vacuum expectation value [the vevs should not be in the charged direction to preserve $U(1)_{\rm QED}$]:

$$\langle\, \Phi\, \rangle_0 \equiv \langle\, 0 |\, \Phi\, |\, 0\, \rangle = \begin{pmatrix} 0 \\ \frac{v}{\sqrt{2}} \end{pmatrix} \quad \text{with} \quad v = \left(-\frac{\mu^2}{\lambda}\right)^{1/2} \tag{2.10}$$

We can make now, the exercise discussed in details in the previous lectures [1]:

– we write the field Φ in terms of four fields $\theta_{1,2,3}(x)$ and $H(x)$ at first order:

$$\Phi(x) = e^{i\theta_a(x)\tau^a(x)/v} \begin{pmatrix} 0 \\ \frac{1}{\sqrt{2}}(v + H(x)\,) \end{pmatrix} \simeq \frac{1}{\sqrt{2}} \begin{pmatrix} \theta_2 + i\theta_1 \\ v + H - i\theta_3 \end{pmatrix} \tag{2.11}$$

– we make a gauge transformation on this field to go to the unitary gauge:

$$\Phi(x) \to e^{-i\theta_a(x)\tau^a(x)}\, \Phi(x) = \begin{pmatrix} 0 \\ \frac{1}{\sqrt{2}}(v + H(x) \end{pmatrix} \tag{2.12}$$

– we then fully develop the term $|D_\mu\Phi)|^2$ of the Lagrangian $\mathcal{L}_S$:

$$
\begin{aligned}
|D_\mu\Phi)|^2 &= \left|\Big(\partial_\mu - ig_1\frac{\tau_a}{2}W_\mu^a - i\frac{g_2}{2}B_\mu\Big)\Phi\right|^2 \\
&= \frac{1}{2}\left|\begin{pmatrix} \partial_\mu - \frac{i}{2}(g_2W_\mu^3+g_1B_\mu) & -\frac{ig_2}{2}(W_\mu^1 - iW_\mu^2) \\ -\frac{ig_2}{2}(W_\mu^1+iW_\mu^2) & \partial_\mu + \frac{i}{2}(g_2W_\mu^3 - g_1B_\mu)\end{pmatrix}\begin{pmatrix} 0 \\ v+H\end{pmatrix}\right|^2 \\
&= \frac{1}{2}(\partial_\mu H)^2 + \frac{1}{8}g_2^2(v+H)^2|W_\mu^1+iW_\mu^2|^2 + \frac{1}{8}(v+H)^2|g_2W_\mu^3 - g_1B_\mu|^2
\end{aligned}
$$

– define the new fields $W_\mu^\pm$ and Z_μ [A_μ is the orthogonal of Z_μ]:

$$
W^\pm = \frac{1}{\sqrt{2}}(W_\mu^1 \mp W_\mu^2)\ ,\ Z_\mu = \frac{g_2W_\mu^3 - g_1B_\mu}{\sqrt{g_2^2+g_1^2}}\ ,\ A_\mu = \frac{g_2W_\mu^3 + g_1B_\mu}{\sqrt{g_2^2+g_1^2}} \tag{2.13}
$$

The equations for the neutral fields define the Weinberg weak mixing angle $\sin\theta_W$

$$
\sin\theta_W = \frac{g_2}{\sqrt{g_1^2+g_2^2}} = \frac{e}{g_2} \tag{2.14}
$$

– and pick up the terms which are bilinear in the fields $W^\pm, Z, A$:

$$
M_W^2 W_\mu^+ W^{-\mu} + \frac{1}{2}M_Z^2 Z_\mu Z^\mu + \frac{1}{2}M_A^2 A_\mu A^\mu \tag{2.15}
$$

The W and Z bosons have acquired masses, while the photon is still massless

$$
M_W = \frac{1}{2}vg_2\ \ ,\ M_Z = \frac{1}{2}v\sqrt{g_2^2+g_1^2}\ \text{ and } M_A = 0 \tag{2.16}
$$

Thus, we have achieved (half of) our goal: by spontaneously breaking the symmetry $\mathrm{SU(2)_L}\times \mathrm{U(1)_Y} \to \mathrm{U(1)_{QED}}$, three Goldstone bosons have bean eaten by the $W^\pm$ and Z boson to get their masses. Since the $\mathrm{U(1)_{QED}}$ symmetry is still unbroken, the photon which is its generator, remains massless.

What about the fermion masses? In fact, we can also generate the fermion masses using the same scalar field Φ (with hypercharge $Y=1$) and the isodoublet $\tilde{\Phi} = i\tau_2\Phi^*$ (with hypercharge $Y=-1$). For any fermion generation, we introduce the SU(2)× U(1) invariant Yukawa Lagrangian:

$$
\mathcal{L}_F = -f_e\,\bar{L}\,\Phi\,e_R - f_d\,\bar{Q}\,\Phi\,d_R - f_u\,\bar{Q}\,\tilde{\Phi}\,u_R\ + \text{h.c.} \tag{2.17}
$$

and repeat the same exercise as previously, i.e. $\Phi^* \to (0, v+H)/\sqrt{2}$ etc..

$$
\mathcal{L}_F = -\frac{1}{\sqrt{2}}f_e\,(\bar{\nu}_e, \bar{e}_L)\begin{pmatrix} 0 \\ v+H\end{pmatrix} e_R\ + \cdots\ = -\frac{1}{\sqrt{2}}\,(v+H)\,\bar{e}_L e_R\ + \cdots \tag{2.18}
$$

The constant term in front of $\bar{e}_L e_R$ (and h.c.) is identified with the fermion mass:

$$m_e = \frac{f_e\, v}{\sqrt{2}} \quad , \; m_u = \frac{f_u\, v}{\sqrt{2}} \quad , \; m_d = \frac{f_d\, v}{\sqrt{2}} \tag{2.19}$$

Thus, with the same isodoublet Φ of scalar fields, we have generated the masses of both the gauge bosons $W^\pm, Z$ and fermions, while preserving the SU(2)×U(1) gauge symmetry (which is now spontaneously broken or hidden).

Finally, let us come to the Higgs boson itself. The kinetic part of the Higgs field, $\frac{1}{2}(\partial_\mu H)^2$, comes from the term involving the covariant derivative $|D_\mu \Phi|^2$, while the mass and self–interaction parts, come from the scalar potential $V(\Phi) = \mu^2 \Phi^\dagger \Phi + \lambda (\Phi^\dagger \Phi)^2$

$$V \;=\; \frac{\mu^2}{2}(0, v+H) \begin{pmatrix} 0 \\ v+H \end{pmatrix} + \frac{\lambda}{2} \left| (0, v+H) \begin{pmatrix} 0 \\ v+H \end{pmatrix} \right|^2 \tag{2.20}$$

and using the relation $v^2 = -\mu^2/\lambda$, one obtains:

$$V \;=\; -\frac{1}{2}\lambda v^2\, (v+H)^2 + \frac{1}{4}\lambda (v+H)^4 \tag{2.21}$$

Doing a simple exercise, one finds that the Lagrangian containing H is given by

$$\mathcal{L}_H = \frac{1}{2}(\partial_\mu H)(\partial^\mu H) - V = \frac{1}{2}(\partial^\mu H)^2 - \lambda v^2\, H^2 - \lambda v\, H^3 - \frac{\lambda}{4}\, H^4 \tag{2.22}$$

from which, one can see that the Higgs boson mass simply reads

$$M_H^2 = 2\lambda v^2 = -2\mu^2 \tag{2.23}$$

while the Feynman rules [recall the discussion given in section 1.1] for the Higgs self–interaction vertices are given by:

$$g_{H^3} = (3!) i \lambda v = 3i\, \frac{M_H^2}{v} \quad , \; g_{H^4} = (4!) i \frac{\lambda}{4} = 3i \frac{M_H^2}{v^2} \tag{2.24}$$

In the case of the Higgs boson couplings to gauge bosons and fermions, they were almost derived previously, when the masses of these particles were calculated. Indeed, from the Lagrangian giving the gauge boson and fermion masses:

$$\mathcal{L}_{M_V} \sim M_V^2 \left(1 + \frac{H}{v}\right)^2 \quad , \; \mathcal{L}_{m_f} \sim -m_f \left(1 + \frac{H}{v}\right) \tag{2.25}$$

one obtains also the Higgs boson couplings to gauge bosons and fermions:

$$g_{Hff} = i \frac{m_f}{v} \quad , \; g_{HVV} = -2i \frac{M_V^2}{v} \quad , \; g_{HHVV} = -2i \frac{M_V^2}{v^2} \tag{2.26}$$

Note that the value of the vacuum expectation value v is fixed in terms of the W boson mass or the Fermi constant G_μ derived precisely from muon decay:

$$M_W = \frac{1}{2} g v = \left(\frac{\sqrt{2} g^2}{8 G_\mu} \right)^{1/2} \Rightarrow v = \frac{1}{(\sqrt{2} G_\mu)^{1/2}} \simeq 246 \text{ GeV} \tag{2.27}$$

2.1.2 The scalar Higgs potential of the MSSM

In the MSSM, we need two doublets of complex scalar fields of opposite hypercharge

$$H_1 = \left(\begin{array}{c} H_1^0 \\ H_1^- \end{array} \right) \text{ with } Y_{H_1} = +1 \quad , \quad H_2 = \left(\begin{array}{c} H_2^+ \\ H_2^0 \end{array} \right) \text{ with } Y_{H_2} = -1 \tag{2.28}$$

to break the electroweak symmetry. There are two reasons for this:

i) In the SM, there are in principle chiral or Adler–Bardeen–Jackiw anomalies which originate from triangular fermionic loops involving axial–vector current couplings and which spoil the renormalisability of the theory. However, these anomalies disappear because the sum of the hypercharges or charges of all the 15 chiral fermions of one generation in the SM is zero

$$\mathrm{Tr}(Y_f) = \mathrm{Tr}(Q_f) = \sum Q_f = 3(2/3) + 3(-1/3) + (-1) = 0 \tag{2.29}$$

In the SUSY case, if we use only one doublet of Higgs fields as in the SM, we will have one additional charged spin 1/2 particle, the higgsino corresponding to the SUSY partner of the charged component of the scalar field, which will spoil this cancellation. With two doublet with opposite hypercharge, the cancellation of chiral anomalies still takes place.

ii) In the SM one generates the fermion masses by using the same scalar field Φ with hypercharge $Y = 1$ which generates the W and Z boson masses, and the isodoublet $\tilde{\Phi} = i\tau_2 \Phi^*$ with hypercharge $Y = -1$. This is performed by the introduction of the SU(2)$\times$ U(1) invariant Yukawa Lagrangian as discussed previously

$$\mathcal{L}_F = -f_e \, \bar{L} \, \Phi \, e_R - f_d \, \bar{Q} \, \Phi \, d_R - f_u \, \bar{Q} \, \tilde{\Phi} \, u_R \; + \text{h.c.} \tag{2.30}$$

However in a SUSY theory, and as discussed in section 1.3, the Superpotential should involve only the superfields and not their conjugates. Therefore we cannot use the same doublet Φ as well as its conjugate $\tilde{\Phi}$ and we must introduce a second doublet with hypercharge $Y = -1$ to generate the masses of the isospin up–type fermions.

In the MSSM, the terms contributing to the scalar Higgs potential V come from three different sources:

i) The D terms containing the quartic Higgs interactions

$$V_D = \frac{1}{2} \sum_a \Big(\sum_i g_a S_i^* T^a S_i \Big)^2 \tag{2.31}$$

For the two Higgs fields H_1, H_2 with $Y = +1, -1$, these D terms are given by

$$\begin{aligned} \mathrm{U(1)_Y} \quad &: \quad V_D^1 = \left[\frac{g_1}{2} (|H_2|^2 - |H_1|^2) \right]^2 \\ \mathrm{SU(2)_L} \quad &: \quad V_D^2 = \left[\frac{g_2}{2} (H_1^{i*} \tau_{ij}^a H_1^j + H_2^{i*} \tau_{ij}^a H_2^j) \right]^2 \end{aligned} \tag{2.32}$$

with $\tau^a = 2T^a$. Using the SU(2) identity $\tau^a_{ij}\tau^a_{kl} = 2\delta_{il}\delta_{jk} - \delta_{ij}\delta_{kl}$, one obtains the potential

$$V_D = \frac{g_2^2}{8}\left[4|H_1^*.H_2|^2 - 2(H_1^*.H_2)(H_2^*.H_2) + (|H_1|^2 + |H_2|^2)\right]^2 + \frac{g_1^2}{8}(|H_2|^2 - |H_1|^2)^2 \quad (2.33)$$

ii) The F term of the Superpotential, which as discussed in section 1.3 can be written as

$$V_F = \sum_i \left|\frac{\partial W(z_i)}{\partial z_i}\right|^2 \rightarrow \sum_i \left|\frac{\partial W(\phi_j)}{\partial \phi_i}\right|^2 \quad (2.34)$$

Simply from $W \sim \mu \hat{H}_1.\hat{H}_2$, one obtains the component of the potential:

$$V_F = \mu^2(|H_1|^2 + |H_2|^2) \quad (2.35)$$

iii) Finally, there is a piece originating from the soft–SUSY breaking scalar Higgs mass terms and the bilinear term

$$V_{\rm soft} = m_1^2 H_1^\dagger H_1 + m_2^2 H_2^\dagger H_2 + B\mu(H_2.H_1 + {\rm h.c.}) \quad (2.36)$$

The full scalar potential involving the Higgs fields is the sum of the three terms:

$$\begin{aligned} V_H &= (|\mu|^2 + m_1^2)|H_1|^2 + (|\mu|^2 + m_2^2)|H_2|^2 - \mu B\epsilon_{ij}(H_1^i H_2^j + {\rm h.c.}) \\ &+ \frac{g_2^2 + g_1^2}{8}(|H_1|^2 - |H_2|^2)^2 + \frac{1}{2}g_2^2|H_1^* H_2|^2 \end{aligned} \quad (2.37)$$

Developing the Higgs fields in terms of their charged and neutral components and defining the mass squared

$$\overline{m}_1^2 = |\mu|^2 + m_1^2\,,\ \overline{m}_2^2 = |\mu|^2 + m_2^2\,,\ \overline{m}_3^2 = B\mu \quad (2.38)$$

one obtains

$$\begin{aligned} V_H &= \overline{m}_1^2(|H_0^1|^2 + |H_1^+|^2) + \overline{m}_2^2(|H_2^0|^2 + |H_2^-|^2) - \overline{m}_3^2(H_1^+ H_2^- - H_1^0 H_2^0 + {\rm hc}) \\ &+ \frac{g_2^2 + g_1^2}{8}(|H_1^0|^2 + |H_1^+|^2 - |H_2^0|^2 - |H_2^-|^2)^2 + \frac{g_2^2}{2}|H_1^{+*} H_1^0 + H_2^{0*} H_2^-|^2 \end{aligned} \quad (2.39)$$

One can then require that the minimum of the potential V_H breaks the $\rm SU(2)_L \times U_Y$ group while preserving the electromagnetic symmetry $\rm U(1)_{QED}$. At the minimum of the potential, $V_H^{\rm min}$ one can always choose the vacuum expectation value of the field H_1^+ to be zero, $\langle H_1^+\rangle$=0, because of SU(2) symmetry. At $\partial V/\partial H_1^+$=0, one obtains then automatically $\langle H_2^-\rangle$=0. There is therefore no breaking in the charged directions and the QED symmetry is preserved. One can then ignore the charged fields H_1^+, H_2^- and, to simplify, write the scalar potential as

$$V_H = \overline{m}_1^2|H_0^1|^2 + \overline{m}_2^2|H_2^0|^2 + \overline{m}_3^2(H_1^0 H_2^0 + {\rm hc}) + \frac{g_2^2 + g_1^2}{8}(|H_1^0|^2 - |H_2^0|^2)^2 \quad (2.40)$$

Some interesting and important remarks can be made at this stage:

- The quartic Higgs couplings are fixed in terms of the $\mathrm{SU}(2) \times \mathrm{U}(1)$ gauge couplings. Contrary to a general non supersymmetric two–Higgs doublet model where the scalar potential V_H has 6 free parameters and a phase, in the MSSM we have only three free parameters: $\overline{m}_1^2, \overline{m}_2^2$ and $\overline{m}_3^2$.
- The two combinations $m_{1,2}^2 + |\mu|^2$ are real, and thus, only $B\mu$ can be complex. However, any phase in $B\mu$ can be absorbed into the phases of the fields H_1, H_2. Thus, the scalar potential of the MSSM is CP conserving.
- If the product $B\mu$ is zero, all other terms in the scalar potential are positive and thus the scalar potential vanishes only if the two vacuum expectation values are zero, $\langle H_1^0 \rangle = \langle H_2^0 \rangle = 0$. Thus, to have electroweak symmetry breaking (without charge and color breaking), we need $\overline{m}_{1,2,3} \neq 0$.
- In the direction $|H_1^0|$=$|H_2^0|$, there is no quartic term. V_H is bounded from below for large values of the field H_i only if the following condition is satisfied:

$$\overline{m}_1^2 + \overline{m}_2^2 > 2|\overline{m}_3^2| \Rightarrow \overline{m}_1^2 = \overline{m}_2^2 \neq 0 \tag{2.41}$$

- To have explicit electroweak symmetry breaking, and thus a negative mass squared term in the Lagrangian, we need to make sure that the potential at the minimum is a saddle point and therefore

$$\mathrm{Det}\Big(\partial^2 V_H / \partial H_i^0 \partial H_j^0\Big) < 0 \Rightarrow \overline{m}_1^2 \overline{m}_2^2 < \overline{m}_3^4 \tag{2.42}$$

- The two above conditions on the masses $\bar{m}_i$ are not satisfied if $\overline{m}_1^2 = \overline{m}_2^2$, and thus we must have non–vanishing soft–SUSY breaking scalar masses m_1 and m_2

$$\overline{m}_1^2 = \overline{m}_2^2 \Rightarrow m_1^2 \neq m_2^2 \tag{2.43}$$

Therefore to break the electroweak symmetry, we need also to break Supersymmetry! This provides a close connection between gauge symmetry breaking and SUSY breaking.

In other words, for the radiative electroweak symmetry breaking to take place, we need in the SM, in an ad hoc way, to make the choice $\mu^2 < 0$. In the MSSM, the soft–SUSY breaking scalar Higgs masses are positive at high–energy, $M_{H_i}^2 > 0$, but the running to lower energies, via the contributions of top/bottom quarks and their SUSY partners [which have potentially large couplings as will be discussed later] in the RGEs, makes that at the electroweak scale one obtains $M_{H_i}^2 < 0$ which triggers the electroweak symmetry breaking: this is the radiative breaking of the symmetry. Thus, electroweak symmetry breaking is more natural and elegant in the MSSM than in the SM.

2.1.3 The physical Higgs spectrum in the MSSM

Let us now determine the Higgs spectrum in the MSSM. The neutral components of the two Higgs fields develop vacuum expectations values

$$\langle H_1^0 \rangle = v_1 \quad , \quad \langle H_2^0 \rangle = v_2 \tag{2.44}$$

Minimizing the scalar potential at the electroweak minimum, $\partial V_H / \partial H_1^0 = \partial V_H / \partial H_2^0 = 0$, one obtains

$$\begin{aligned} \left.\frac{\partial V_H}{\partial H_1^0}\right|_{\min} &= 2\bar{m}_1^2 v_1 + 2\bar{m}_3^2 v_2 + \frac{1}{8}(g_2^2+g_1^2) \times 2v_1(v_1^2 - v_2^2) \\ \left.\frac{\partial V_H}{\partial H_2^0}\right|_{\min} &= 2\bar{m}_2^2 v_2 - 2\bar{m}_3^2 v_1 + \frac{1}{8}(g_2^2+g_1^2) \times 2v_2(v_2^2 - v_1^2) \end{aligned} \tag{2.45}$$

Using the relation derived already in the SM case,

$$(v_1^2 + v_2)^2 = v^2 = \frac{2M_Z^2}{g_2^2+g_1^2} = (246\ \text{GeV})^2 \tag{2.46}$$

and defining the important parameter

$$\tan\beta = \frac{v_2}{v_1} = \frac{(v\sin\beta)}{(v\cos\beta)} \tag{2.47}$$

one obtains for the two minimization conditions

$$\bar{m}_1^2 = -\bar{m}_3^2\tan\beta - \frac{1}{2}M_Z^2\cos(2\beta) \ , \ \bar{m}_2^2 = -\bar{m}_3^2\text{cot}\beta + \frac{1}{2}M_Z^2\cos(2\beta) \tag{2.48}$$

Alternatively, one can write these conditions in the following way:

$$B\mu = \frac{(m_1^2 - m_2^2)\tan 2\beta + M_Z^2 \sin 2\beta}{2} \ , \ \mu^2 = \frac{m_2^2\sin^2\beta - m_1^2\cos^2\beta}{\cos 2\beta} - \frac{M_Z^2}{2} \tag{2.49}$$

This relations show that if m_1 and m_2 are known, for instance if they are given by the RGEs at the weak scale once they are are fixed to a given value of the GUT scale, together with the knowledge of the value of $\tan\beta$, this fixes the values of the parameters B and μ^2. The sign of μ stays undetermined. These relations are very important since in constrained models, such as mSUGRA for instance, with the requirement of radiative symmetry breaking, it leads to additional constraints on the number of free parameters.

To obtain the Higgs physical fields and their masses, one has to develop the two doublet complex scalar fields H_1 and H_2 [i.e. 4+4 degrees of freedom] around the vacuum, into real and imaginary parts:

$$\begin{aligned} H_1 = (H_1^0, H_1^-) &= \left(v_1 + \frac{1}{\sqrt{2}}(H_1^0 + iP_1^0) \ , \ \frac{1}{\sqrt{2}}(H_1^- + iP_1^-) \right) \\ H_2 = (H_2^+, H_2^0) &= \left(\frac{1}{\sqrt{2}}(H_2^+ - iP_2^+) \ , \ v_2 + \frac{1}{\sqrt{2}}(H_2^0 + iP_2^0)/\sqrt{2} \right) \end{aligned} \tag{2.50}$$

where the real parts correspond to the CP–even and charged Higgs bosons and the imaginary parts corresponds to the CP–odd Higgs and the Goldstones bosons, and then diagonalize the mass matrices evaluated at the vacuum

$$\mathcal{M}^2_{ij} = \frac{1}{2} \left. \frac{\partial^2 V_H}{\partial H_i \partial H_j} \right|_{\langle H_1^0 \rangle = v_1, \langle H_2^0 \rangle = v_2, \langle H_{1,2}^\pm \rangle = 0} \tag{2.51}$$

To obtain the Higgs boson masses and their mixing angles, two useful relations are as follows

$$\begin{gathered} \mathrm{Tr}(\mathcal{M}^2) = M_1^2 + M_2^2 \quad , \quad \mathrm{Det}(\mathcal{M}^2) = M_1^2 M_2^2 \\ \sin 2\theta = \frac{2\mathcal{M}_{12}}{\sqrt{(\mathcal{M}_{11} - \mathcal{M}_{22})^2 + 4\mathcal{M}_{12}^2}} \ , \ \cos 2\theta = \frac{\mathcal{M}_{11} - \mathcal{M}_{22}}{\sqrt{(\mathcal{M}_{11} - \mathcal{M}_{22})^2 + 4\mathcal{M}_{12}^2}} \end{gathered} \tag{2.52}$$

where M_1 and M_2 are the physical masses and θ the mixing angle.

The procedure in the case of the CP–even Higgs bosons is as follows. Dropping the subscripts for simplicity, the neutral part of the scalar potential is given by:

$$V_H = \overline{m}_1^2 |H_1|^2 + \overline{m}_2^2 |H_2|^2 + \overline{m}_3^2 (H_1 H_2 + \mathrm{hc}) + \frac{M_Z^2}{4v^2} (|H_1|^2 - |H_2|^2)^2 \tag{2.53}$$

One first performs the first derivative of the scalar potential:

$$\begin{aligned} \frac{\partial V_H}{\partial H_1^0} &= 2\bar{m}_1^2 H_1 + 2\bar{m}_3^2 H_2 + \frac{M_Z^2}{v^2} H_1 (H_1^2 - H_2^2) \\ \frac{\partial V_H}{\partial H_2^0} &= 2\bar{m}_2^2 H_2 + 2\bar{m}_3^2 H_1 + \frac{M_Z^2}{v^2} H_2 (H_2^2 - H_1^2) \end{aligned} \tag{2.54}$$

At the minimum, one has $\partial V_H / \partial H_{1,2} = 0$, leading to the two relations for $\bar{m}_1^2$ and $\bar{m}_2^2$ discussed above. One then performs the second derivative with respect to H_1 and $H2$:

$$\begin{aligned} \frac{\partial^2 V_H}{\partial H_1^0 \partial H_1^0} &= 2\bar{m}_1^2 + \frac{M_Z^2}{v^2} (H_1^2 - H_2^2 + 2H_1 H_1) = 2\bar{m}_1^2 + M_Z^2 (3c_\beta^2 - s_\beta^2) \\ \frac{\partial^2 V_H}{\partial H_2^0 \partial H_2^0} &= 2\bar{m}_2^2 + \frac{M_Z^2}{v^2} (H_2^2 - H_1^2 + 2H_2 H_2) = 2\bar{m}_2^2 + M_Z^2 (3s_\beta^2 - c_\beta^2) \\ \frac{\partial^2 V_H}{\partial H_1^0 \partial H_2^0} &= 2\bar{m}_3^2 + \frac{M_Z^2}{v^2} (-2H_1 H_2) = 2\bar{m}_3^2 - M_Z^2 \sin 2\beta \end{aligned}$$

Using the previous relations for $\bar{m}_1$ and $\bar{m}_2$ in terms of $\bar{m}_3$ and M_Z, one then obtains the mass matrix for the CP even Higgs bosons

$$\mathcal{M}_R^2 = \begin{bmatrix} -\bar{m}_3^2 \tan\beta + M_Z^2 \cos^2\beta & \bar{m}_3^2 - M_Z^2 \sin\beta\cos\beta \\ \bar{m}_3^2 M_Z^2 \sin\beta\cos\beta & -\bar{m}_3^2 \mathrm{cot}\beta + M_Z^2 \sin^2\beta \end{bmatrix}$$

In the case of the CP–odd Higgs boson, one can use the same expressions for $\partial^2 V/\partial H_i^0 \partial H_j^0$ as above but setting the fields to zero at the minimum of the potential [remember that now the term $(|H_1|^2 - |H_2|^2)$ has both real and imaginary parts and will also contribute]:

$$
\begin{aligned}
\left.\frac{\partial^2 V_H}{\partial P_1^0 \partial P_1^0}\right|_{\min} &= 2\bar{m}_1^2 + \frac{M_Z^2}{v^2}(H_1^2 - H_2^2) = -2\bar{m}_3^2 \tan\beta \\
\left.\frac{\partial^2 V_H}{\partial P_2^0 \partial P_2^0}\right|_{\min} &= 2\bar{m}_2^2 - \frac{M_Z^2}{v^2}(H_1^2 - H_2^2) = -2\bar{m}_3^2 \cot\beta \\
\left.\frac{\partial^2 V_H}{\partial P_1^0 \partial P_2^0}\right|_{\min} &= 2\bar{m}_3^2
\end{aligned} \tag{2.55}
$$

Leading to the mass matrix for the Goldstone and CP–odd Higgs bosons:

$$
\mathcal{M}_I^2 = \left[\begin{array}{cc} -\bar{m}_3^2 \tan\beta & \bar{m}_3^2 \\ \bar{m}_3^2 & -\bar{m}_3^2 \cot\beta \end{array} \right] \tag{2.56}
$$

Since $\mathrm{Det}\mathcal{M}_I^2 = 0$, one eigenvalue is zero and corresponds to the mass of the Goldstone boson, while the other eigenvalue corresponds to the mass of the pseudoscalar Higgs boson and is given by

$$
M_A^2 = -\bar{m}_3^2(\tan\beta + \cot\beta) = -\frac{2\bar{m}_3^2}{\sin 2\beta} \tag{2.57}
$$

The mixing angle θ, which gives the physical fields is in fact, just the angle β:

$$
\begin{aligned}
\sin 2\theta &= 2\left[\left(\frac{s_\beta}{c_\beta} - \frac{c_\beta}{s_\beta}\right)^2 + 4\right]^{-\frac{1}{2}} = 2\left[\frac{c_{2\beta}^2}{s_{2\beta}^2/4} + 4\right]^{-\frac{1}{2}} = s_{2\beta} \\
\cos 2\theta &= \left(-\frac{s_\beta}{c_\beta} + \frac{c_\beta}{s_\beta}\right)\left[\cdots\cdots\right]^{-1/2} = \left(\frac{c_{2\beta}}{s_{2\beta}}\right)\left[s_{2\beta}\right] = c_{2\beta}
\end{aligned}
$$

with $s_\beta = \sin\beta$ etc... and thus

$$
\left(\begin{array}{c} G^0 \\ A \end{array} \right) = \left(\begin{array}{cc} \cos\beta & \sin\beta \\ -\sin\beta & \cos\beta \end{array} \right) \left(\begin{array}{c} P_1^0 \\ P_2^0 \end{array} \right) \tag{2.58}
$$

In the case of the charged Higgs boson, one can make exactly the same exercise as for the pseudoscalar A boson and obtains the charged fields

$$
\left(\begin{array}{c} G^\pm \\ H^\pm \end{array} \right) = \left(\begin{array}{cc} \cos\beta & \sin\beta \\ -\sin\beta & \cos\beta \end{array} \right) \left(\begin{array}{c} H_1^\pm \\ H_2^\pm \end{array} \right)
$$

with a massless charged Goldstone and a charged Higgs boson with mass

$$
M_{H^\pm}^2 = M_A^2 + M_W^2
$$

Coming back to the CP–even Higgs boson case, and injecting, the expression of M_A^2 into $\mathcal{M}_R^2$, we obtain for the CP–even mass matrix

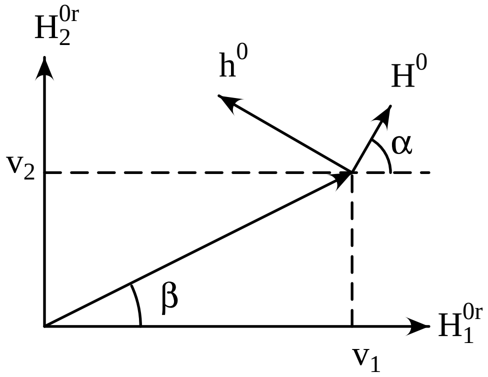
$$\mathcal{M}_R^2 = \begin{bmatrix} M_A^2 s_\beta^2 + M_Z^2 c_\beta^2 & -(M_A^2+M_Z^2)s_\beta c_\beta \\ -(M_A^2+M_Z^2)s_\beta c_\beta & M_A^2 c_\beta^2 + M_Z^2 s_\beta^2 \end{bmatrix} \tag{2.59}$$

Calculating the determinant and the trace, one obtains for the masses:

$$\begin{aligned} \mathrm{Det}\mathcal{M}_R^2 &= (M_A^2 s_\beta^2 + M_Z^2 c_\beta^2)(M_A^2 c_\beta^2 + M_Z^2 s_\beta^2) - (M_A^2+M_Z^2)s_\beta^2 c_\beta^2 = M_A^2 M_Z^2 \cos^2 2\beta \equiv M_h^2 M_H^2 \\ \mathrm{Tr}\mathcal{M}_R^2 &= M_A^2 s_\beta^2 + M_Z^2 c_\beta^2 + M_A^2 c_\beta^2 + M_Z^2 s_\beta^2 = M_A^2 + M_Z^2 \equiv M_h^2 + M_H^2 \end{aligned} \tag{2.60}$$

The masses of the physical CP–even Higgs bosons are obtained from the rotation of angle α [see below]

$$\begin{pmatrix} H \\ h \end{pmatrix} = \begin{pmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{pmatrix} \begin{pmatrix} H_1^0 \\ H_2^0 \end{pmatrix} \tag{2.61}$$



One has to solve the quadratic equation

$$M_h^2(M_A^2 + M_Z^2 - M_h^2) = M_A^2 M_Z^2 \cos^2 2\beta \Rightarrow M_h^4 - M_h^2(M_A^2+M_Z^2) + M_A^2 M_Z^2 \cos^2 2\beta = 0 \tag{2.62}$$

with discriminant $\Delta = (M_A^2 + M_Z^2)^2 - 4M_A^2 M_Z^2 \cos^2 2\beta$, the two solutions are: $M_{h,H}^2 = (M_A^2 + M_Z^2 \mp \sqrt{\Delta})/2$ giving the masses of the physical H and h bosons, where h is by convention the lightest of the two particles:

$$M_{h,H}^2 = \frac{1}{2}\left[M_A^2 + M_Z^2 \mp \sqrt{(M_A^2+M_Z^2)^2 - 4M_A^2 M_Z^2 \cos^2 2\beta}\right] \tag{2.63}$$

The mixing angle α which rotates the fields is given by $(-\frac{\pi}{2} \le \alpha \le 0)$

$$\tan 2\alpha = \frac{2\mathcal{M}_{12}}{\mathcal{M}_{11} - \mathcal{M}_{22}} = \frac{-(M_A^2+M_Z^2)\sin 2\beta}{(M_Z^2 - M_A^2)\cos 2\beta} = \tan 2\beta \, \frac{M_A^2 + M_Z^2}{M_A^2 - M_Z^2} \tag{2.64}$$

2.2 The masses and couplings of the MSSM Higgs bosons

2.2.1 The Higgs boson couplings at the tree level

Trilinear and Quartic scalar couplings:

The trilinear (i.e. the couplings between three Higgs fields) and the quartic (couplings between four Higgs fields) couplings among Higgs bosons can be obtained from the scalar potential V_H by making

$$\begin{aligned}
\lambda^2_{ijk} &= \frac{\partial^3 V_H}{\partial H_i \partial H_j \partial H_k}\Big|_{\langle H_1^0\rangle = v_1, \langle H_2^0\rangle = v_2, \langle H_{1,2}^{\pm}\rangle = 0} \\
\lambda^2_{ijkl} &= \frac{\partial^4 V_H}{\partial H_i \partial H_j \partial H_k \partial H_l}\Big|_{\langle H_1^0\rangle = v_1, \langle H_2^0\rangle = v_2, \langle H_{1,2}^{\pm}\rangle = 0}
\end{aligned} \tag{2.65}$$

with the H_i expressed in terms of the fields $h, H, A, H^{\pm}$ and $G^0, G^{\pm}$ with the rotations of angles β et α discussed in the previous section. The various trilinear couplings, in units of $\lambda_0 = -iM_Z^2/v$, are given by

$$\begin{aligned}
\lambda_{hhh} &= 3\cos 2\alpha \sin(\beta+\alpha) \\
\lambda_{HHH} &= 3\cos 2\alpha \cos(\beta+\alpha) \\
\lambda_{Hhh} &= 2\sin 2\alpha \sin(\beta+\alpha) - \cos 2\alpha \cos(\beta+\alpha) \\
\lambda_{HHh} &= -2\sin 2\alpha \cos(\beta+\alpha) - \cos 2\alpha \sin(\beta+\alpha) \\
\lambda_{HAA} &= -\cos 2\beta \cos(\beta+\alpha) \\
\lambda_{hAA} &= \cos 2\beta \sin(\beta+\alpha) \\
\lambda_{HH^+H^-} &= -\cos 2\beta \cos(\beta+\alpha) + 2c_W^2 \cos(\beta-\alpha) \\
\lambda_{hH^+H^-} &= \cos 2\beta \sin(\beta+\alpha) + 2c_W^2 \sin(\beta-\alpha)
\end{aligned} \tag{2.66}$$

While the quartic Higgs couplings are more numerous, an important one is:

$$\lambda_{hhhh} = 3\cos^2\alpha / M_Z^2 \quad (\text{in units of } \lambda_0^2) \tag{2.67}$$

Couplings to gauge bosons:

The Higgs couplings to massive gauge bosons are obtained from the kinetic terms of the fields H_1 et H_2 in the $\mathrm{SU(2)_L \times U(1)_Y}$ Lagrangian:

$$\mathcal{L}_{\text{kin.}} = (D^\mu H_1)^\dagger (D_\mu H_1) + (D^\mu H_2)^\dagger (D_\mu H_2) \tag{2.68}$$

One develops the covariant derivative D_μ and makes the usual transformations on the fields:

$$D_\mu = i\partial_\mu - g\frac{\vec{\tau}_a}{2} W^a_\mu - g'\frac{Y_{H_i}}{2} B_\mu$$

$$W_\mu^\pm = \frac{W_\mu^1 - iW_\mu^2}{\sqrt{2}}, A_\mu = \frac{g' W_\mu^2 + gB_\mu}{g^2 + g'^2}, Z_\mu = \frac{gW_\mu^2 - g'B_\mu}{g^2 + g'^2}$$
$$H_1, H_2 \to h, H, A, H^\pm \;\; \text{et} \;\; G^0, G^\pm \;\; \text{via rotations}\, \beta, \alpha \tag{2.69}$$

One identifies the couplings among Higgs and gauge bosons as

$$\begin{aligned} g_{h_iVV} &\equiv \text{coefficients de}\, h_i V_\mu V_\mu \;\; (g_{\mu\nu}) \\ g_{h_ih_jV} &\equiv \text{coefficients de}\, h_i h_j V_\mu \;\; (\partial_\mu \to p_\mu) \\ g_{h_ih_jVV} &\equiv \text{coefficients de}\, h_i h_j V_\mu V_\mu \;\; (g_{\mu\nu}) \end{aligned} \tag{2.70}$$

Some very important couplings for the phenomenology of Higgs bosons are

$$\begin{aligned} Z^\mu Z^\nu h \;:\; \frac{igM_Z}{\cos\theta_W}\sin(\beta-\alpha)g^{\mu\nu} \quad &, \quad Z^\mu Z^\nu H \;:\; \frac{igM_Z}{\cos\theta_W}\cos(\beta-\alpha)g^{\mu\nu} \\ W^\mu W^\nu h \;:\; igM_W \sin(\beta-\alpha)g^{\mu\nu} \quad &, \quad W^\mu W^\nu H \;:\; igM_W \cos(\beta-\alpha)g^{\mu\nu} \\ Z^\mu hA : \frac{g\cos(\beta-\alpha)}{2\cos\theta_W}(p+p')^\mu \quad &, \quad Z^\mu HA : -\frac{g\sin(\beta-\alpha)}{2\cos\theta_W}(p+p')^\mu \end{aligned} \tag{2.71}$$

A few remarks can be made here: (i) Since the photon is massless, it has no coupling to the neutral Higgs bosons at the tree–level. (ii) CP invariance implies that there are no ZZA and Zhh, ZHh, ZHH couplings for instance. (ii) The couplings of the CP–even Higgs bosons h and H complementary: $g_{hZZ}^2 + g_{HZZ}^2 = g_{\rm MS}^2$.

Yukawa couplings to fermions:

The Higgs boson couplings to fermions originate from the Superpotential W

$$W = \sum_{\rm i,j=gen} Y_{ij}^u \, \hat{u}_R^i \hat{H}_2.\hat{Q}^j + Y_{ij}^d \, \hat{d}_R^i \hat{H}_1.\hat{Q}^j + Y_{ij}^l \, \hat{l}_R^i \hat{H}_1.\hat{L}^j + \mu \hat{H}_1.\hat{H}_2 \tag{2.72}$$

with

$$\mathcal{L}_{\rm Yuk} = -\frac{1}{2}\sum_{ij}\left[\overline{\psi}_{iL}\frac{\partial^2 W}{\partial z_i \partial z_j}\psi_j + \text{h.c.}\right] \tag{2.73}$$

evaluated in terms of the scalar fields $H_{1,2}$. Discarding the bilinear terms, and assuming diagonal Y matrices which are related to the fermion masses, and expressing the fields H_1, H_2 in terms of physical fields, one obtains the Yukawa Lagrangian:

$$\begin{aligned} \mathcal{L}_{\rm Yuk} \;\; = \;\; & -\frac{gm_u}{2M_W \sin\beta}\left[\bar{u}u(H\sin\alpha + h\cos\alpha) - i\bar{u}\gamma_5 u\, A\cos\beta\right] \\ & -\frac{gm_d}{2M_W\cos\beta}\left[\bar{d}d(H\cos\alpha - h\sin\alpha) - i\bar{d}\gamma_5 d\, A\sin\beta\right] \\ & +\frac{g}{2\sqrt{2}M_W}\left\{H^+\bar{u}[m_d\tan\beta(1+\gamma_5) + \frac{m_u}{\tan\beta}(1+\gamma_5)]d + \text{hc}\right\} \end{aligned} \tag{2.74}$$

The MSSM Higgs boson couplings to fermions, in terms of those of the SM Higgs boson $H_{\rm SM}$ [with a normalization factor $-(i)gm_f/2M_W = -im_f/v$] are shown in the table below.

f	g_{ffh}	g_{ffH}	g_{ffA}
u	$\cos\alpha/\sin\beta$	$\sin\alpha/\sin\beta$	$\cot\beta$
d	$-\sin\alpha/\cos\beta$	$\cos\alpha/\cos\beta$	$\tan\beta$

One can notice that: (i) The couplings of the $H^\pm$ bosons have the same intensity as those of the pseudoscalar A boson. (i) For values of $\tan\beta > 1$: the Higgs couplings to isospin down–type fermions are enhanced, while the couplings to up–type fermions are suppressed. (iii) For large values of $\tan\beta$, the Higgs boson couplings to b quarks, proportional to $m_b \tan\beta$, become very strong.

Couplings to the SUSY particles:

The MSSM Higgs boson couplings to scalar fermions come from three different sources:
– The F terms due to the superpotential W (trilinear terms).
– The D terms D due to the kinetic part of the sfermions in $\mathcal{L}$.
– The Lagrangian $\mathcal{L}_{\rm tril.}$ which softly breaks SUSY.

[Note that the scalar masses come directly from the soft–SUSY breaking potential $\mathcal{L}_{\rm soft}$]. These couplings are thus rather complicated in general. An example is given by the of the coupling of the lightest Higgs boson h to the lightest top squarks which reads

$$g_{h\tilde{t}_1\tilde{t}_1} = \frac{M_Z}{c_W}\sin(\beta+\alpha)\left[I_3^t c^2_{\theta_{\tilde{t}}} - Q_t s_W^2 c_{2\theta_{\tilde{t}}}\right] - \frac{m_t^2}{M_W}\frac{\cos\alpha}{\sin\beta} - \frac{m_t s_{2\theta_{\tilde{t}}}}{2M_W}\left[A_t\frac{\cos\alpha}{\sin\beta} + \mu\frac{\sin\alpha}{\sin\beta}\right] \quad (2.75)$$

The coupling is thus potentially very large since it involves terms $\propto m_t^2$ and $m_t A_t$.

The Higgs boson couplings to neutralinos and charginos come from several sources: the F terms, the D terms, the superpotential [in particular from the bilinear term], the gaugino masses in $\mathcal{L}_{\rm soft}$. They are made more complicated by the higgsinos–gauginos mixing, the diagonalization of the gaugino mass matrices, and the Majorana nature of the neutralinos. An example is the lightest Higgs boson coupling to neutralinos $g_{h\chi_i^0\chi_j^0}$, which when normalized to $ig/4$ reads

$$\begin{aligned}
g_{h\chi_i^0\chi_j^0} &\sim (Q_{ij}^*\sin\alpha - S_{ij}^*\cos\alpha)(1-\gamma_5) + (Q_{ij}\sin\alpha + S_{ij}\cos\alpha)(1+\gamma_5)\\
S_{ij} &= Z_{i3}(Z_{j2} - Z_{j1}\tan\theta_W) + Z_{j3}(Z_{i2} - Z_{i1}\tan\theta_W)\\
R_{ij} &= Z_{i4}(Z_{j2} - Z_{j1}\tan\theta_W) + Z_{j4}(Z_{i2} - Z_{i1}\tan\theta_W) \qquad (2.76)
\end{aligned}$$

where Z is the 4×4 matrix which diagonalizes the neutralino matrix $\mathcal{M}_N = f(M_1, M_2, \mu, \tan\beta)$. For the Higgs couplings to the χ_1^0 LSP for which the Z_{11}, Z_{12} are the gaugino components and Z_{13}, Z_{13} are the higgsino components, the coupling vanishes if the LSP is a pure gaugino or a pure higgsino. The h boson couples thus to a higgsino-gaugino mixture.

2.2.2 The decoupling limit and the radiative corrections

Let us recall the expressions of the Higgs masses and mixing angle α which have been derived in the previous section in terms of the pseudoscalar mass M_A and the ratio of vevs $\tan\beta$:

$$\begin{aligned}
M_{h,H}^2 &= \frac{1}{2}\left[M_A^2 + M_Z^2 \mp \sqrt{(M_A^2+M_Z^2)^2 - 4M_A^2 M_Z^2 \cos^2 2\beta}\right] \\
M_{H^\pm}^2 &= M_A^2 + M_W^2 \\
\tan 2\alpha &= \tan 2\beta \, \frac{M_A^2+M_Z^2}{M_A^2-M_Z^2} \quad \text{with} \quad -\frac{\pi}{2} \leq \alpha \leq 0
\end{aligned} \tag{2.77}$$

We see that we have a very important constraint on the lightest h boson

$$M_h \;\leq\; \min(M_A, M_Z)\cdot|\cos 2\beta| \leq M_Z \tag{2.78}$$

besides some other relations: $M_H > \max(M_A, M_Z)$ and $M_{H^\pm}^2 > M_W$. If we send M_A to infinity, we will have the following relations for the Higgs boson masses and α:

$$\begin{gathered}
M_h \sim M_Z \cos 2\beta \; , \; M_H \sim M_{H^\pm} \sim M_A \\
\cos 2\alpha \sim -\cos 2\beta \; , \; \sin 2\alpha \sim -\sin 2\beta \; \Rightarrow \alpha \sim \frac{\pi}{2} - \beta
\end{gathered} \tag{2.79}$$

This is the decoupling regime, where all the Higgs bosons are heavy and degenerate in mass, except for the lightest h which should be lighter than the Z boson. Such a Higgs particle should have been seen at LEP2 since it is kinematically accessible [see next section]: $\sqrt{s}^{\rm max}_{\rm LEP2} \sim 209$ GeV $> M_Z + M_h \gtrsim 184$ GeV! So what happened in this case? Could it be that the MSSM is ruled out?

The answer is no. The previous relation holds only at first order, i.e. at the tree–level. However, as discussed previously, very strong couplings are involved in the theory, in particular the h couplings to top quarks and top squarks. The calculation of radiative corrections to M_h is therefore necessary and must be performed.

In the radiative corrections to the h boson mass, one has to include the important one–loop corrections due to top (s)quarks. In addition to the two–point functions including fermion and scalar loops that have been discussed in the previous section, one has also tadpole contributions which correspond to the counterterm corrections in the MSSM. Their Feynman diagrams are shown below.

To calculate these corrections, let us make the following simplifications: we take the decoupling limit $M_A \to 0$ and large $\tan\beta$ values $\tan\beta \gg 1$ to maximize the h boson mass [we are interesting in the upper bound on M_h]. We then assume that there is no mixing between the two stop squarks and thus, set $m_{\tilde{t}_1} = m_{\tilde{t}_2} = m_{\tilde{t}}$. In this case, the Higgs boson couplings to these particles are particularly simple and are given by: $h\bar{t}t \sim h\tilde{\bar{t}}\tilde{t} \sim \lambda_t$, $hh\tilde{t}^*\tilde{t} \sim \lambda_t^2$ with $\lambda_t = \sqrt{2}m_t/v$. To simplify the algebra we will also assume that the Higgs boson is much lighter than the top quark and squarks, $M_h \ll m_t, m_{\tilde{t}}$.

The calculation of the radiative corrections to M_h is in fact almost already done. For the contributions of the quark and squark two–point functions we already had:

$$\Delta M_h^2|_{\rm 2PF} = \frac{3\lambda_t^2}{4\pi^2}\left[(m_t^2 - m_{\tilde{t}}^2)\log\left(\frac{\Lambda}{m_{\tilde{t}}}\right) + 3m_t^2\log\left(\frac{m_{\tilde{t}}}{m_t}\right)\right] \tag{2.80}$$

For the tadpole contributions, the calculation is extremely simple:

$$\begin{aligned}\Delta M_h^2|_{\rm 1PF} &= i\left(\frac{-iM_H^2}{v}\right)\frac{i}{-M_h^2}\left(-\frac{\lambda_f}{\sqrt{2}}\right)(-4mi)\int\frac{{\rm d}^4k}{(2\pi)^4}\frac{1}{k^2-m_f^2} \\ &+ i\left(\frac{-iM_H^2}{v}\right)\frac{i}{-M_h^2}(iv\lambda_S)i\int\frac{{\rm d}^4k}{(2\pi)^4}\frac{1}{k^2-m_S^2} \\ &= \frac{4N_f m_f\lambda_f}{\sqrt{2}v16\pi^2}\int_0^{\Lambda^2}dy\frac{y}{y+m_f^2}\frac{2N_S\lambda_S}{16\pi^2}\int_0^{\Lambda^2}dy\frac{y}{y+m_S^2}\end{aligned} \tag{2.81}$$

Using the coupling relations $\lambda_S = -\lambda_f^2 = -2m_f^2/v$ and performing the integrals which give $\int ... = \Lambda^2 - m^2\log(\Lambda^2/m^2)$, one finally obtains

$$\Delta M_h^2|_{\rm 1PF} = -\frac{3\lambda_t^2}{4\pi^2}\left[m_{\tilde{t}}^2\log\left(\frac{\Lambda}{m_{\tilde{t}}}\right) - m_t^2\log\left(\frac{\Lambda}{m_{\tilde{t}}}\right)\right] \tag{2.82}$$

Again, the result when fermion and sfermion loops are added does not have quadratic [contrary to the separate contribution]. The total correction to the h boson mass, when the one and two–point functions contributions are added, and using the relation $v = (\sqrt{2}G_\mu)^{-1/2}$, is then given by:

$$\Delta M_h^2 = = \frac{3G_\mu}{\sqrt{2}\pi^2}m_t^4\log\frac{m_{\tilde{t}}^2}{m_t^2} \tag{2.83}$$

As can be seen, the correction grows with the quartic top quark mass, $\Delta M_h^2 \propto m_t^4$ and logarithmically with the stop masses, $\Delta M_h^2 \propto \log(m_{\tilde{t}}^2/m_t^2)$. It is therefore very large and increases the h boson mass by several 10 GeV, $M_h^{\rm max} \to M_Z + 40$ GeV. This explains why the h boson has not been seen at LEP2: the upper bound on the value of M_h in the MSSM, when the one–loop radiative corrections are included, is such that the h can be kinematically not accessible at LEP2 energies.

2.2.3 The Higgs masses and couplings including radiative corrections

Let us discuss more closely the radiative corrections to the Higgs boson masses and couplings in the MSSM. Before the decoupling regime is reached, i.e. when the A boson is rather light, and for not too large values of $\tan\beta$, the expression for the radiative corrections are rather involved: the mixing in the stop sector has to be included as well as the one–loop contributions for the bottom sector. In addition, the two–loop corrections due to QCD corrections and top [and bottom] Yukawa coupling corrections turn out to be rather important and should be included. More discussions can be found in Ref. [5]. However, there is still a rather simple analytical expression which approximate the Higgs boson masses. The main radiative correction, i.e. the one proportional to m_t^4, can parameterized with the quantity ϵ:

$$\epsilon = \frac{3G_F\overline{m}_t^4}{\sqrt{2}\pi^2\sin^2\beta}\left[\log\left(\frac{M_S^2}{m_t^2}\right) + \frac{A_t^2}{M_S^2}\left(1-\frac{A_t^2}{12M_S^2}\right)\right] \tag{2.84}$$

where $\overline{m}_t$ is the running $\overline{\rm MS}$ top quark mass to account for the leading QCD corrections and A_t is the stop trilinear coupling. The Higgs boson masses can be approximated to an accuracy of the order of a few percent compared to the complete result [in particular in the case where the splitting between the two stop masses, and to a lesser extent sbottom masses, is not very large], as functions of M_A, $\tan\beta$ and ϵ with the expression

$$M_{h,H}^2 = \frac{1}{2}(M_A^2+M_Z^2+\epsilon)\left[1\mp\sqrt{1-4\frac{M_Z^2M_A^2\cos^2 2\beta+\epsilon(M_A^2\sin^2\beta+M_Z^2\cos^2\beta)}{(M_A^2+M_Z^2+\epsilon)^2}}\right] \tag{2.85}$$

This radiative correction pushes the maximum value of the lightest h boson mass upward from M_Z by several tens of GeV: in the so–called maximal mixing scenario $\tilde{A}_t = A_t - \mu/\tan\beta = \sqrt{6}M_S$ and with M_A and M_S of about 1 TeV, one obtains an upper mass bound, $M_h \lesssim 130$ GeV for $\tan\beta \gg 1$. In the no mixing scenario, $\tilde{A}_t = 0$, the maximal value of M_h is lower by 20 GeV or so.

In the case of the charged Higgs boson mass, one can also derive a very simple expression for the radiative corrections which gives a result that is rather accurate:

$$M_{H^\pm} = \sqrt{M_A^2+M_W^2-\epsilon_+} \;\; \text{with} \;\; \epsilon_+ = \frac{3G_FM_W^2}{4\sqrt{2}\pi^2}\left[\frac{\overline{m}_t^2}{\sin^2\beta}+\frac{\overline{m}_b^2}{\cos^2\beta}\right]\log\left(\frac{M_S^2}{m_t^2}\right) \tag{2.86}$$

where $\overline{m}_b$ is the running $\overline{\rm MS}$ b mass at the scale of the top quark mass. Notice that in this case, the leading corrections goes as m_t^2 and not m_t^4 and is therefore small.

The masses of the neutral and charged Higgs bosons, including the full set of radiative corrections are shown in Fig. 1. As can be seen, the maximum value for M_h is obtained for the maximal mixing scenario and for large $\tan\beta$ and M_A values. For large M_A values, the A, H and $H^\pm$ bosons are degenerate in mass.

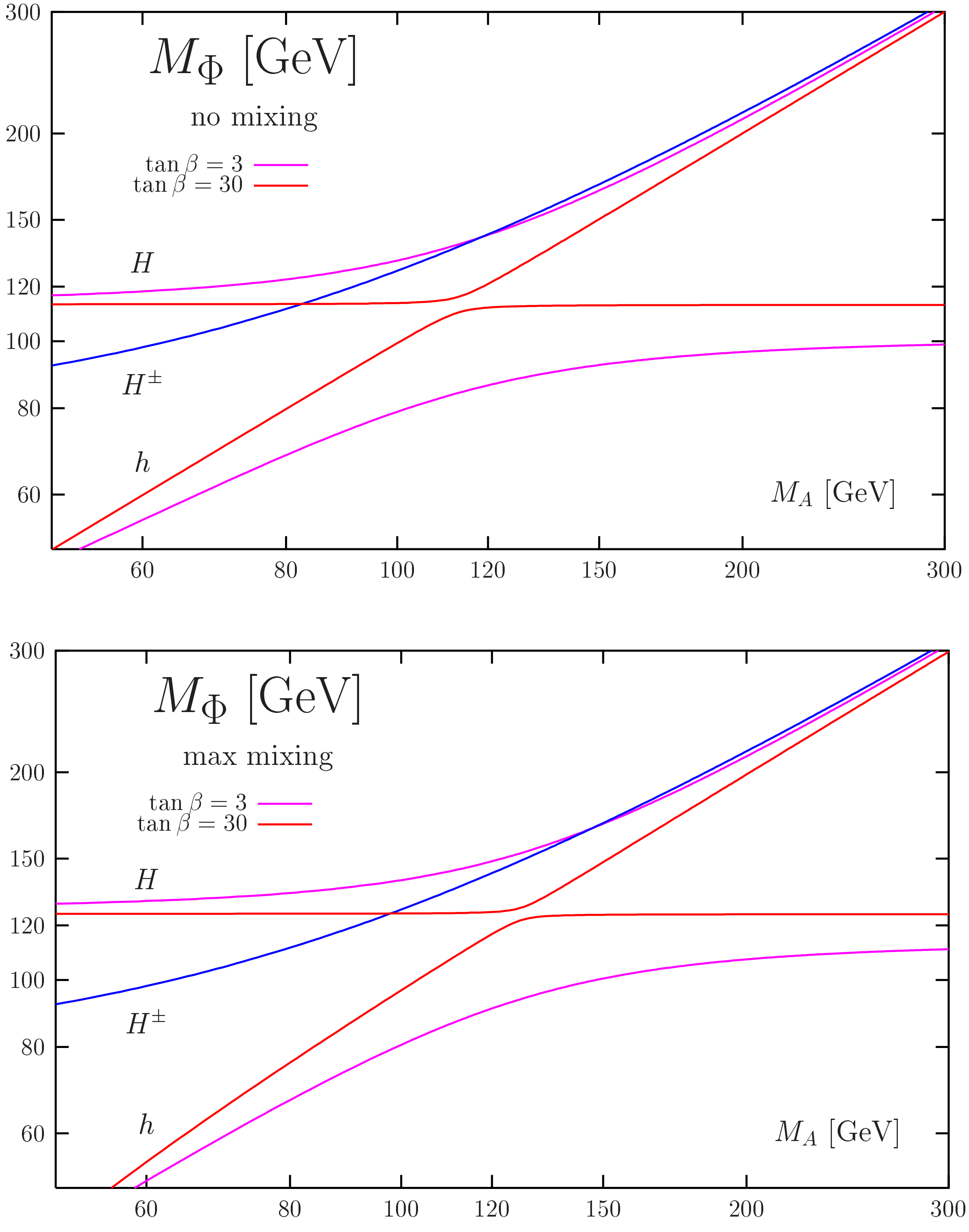


Figure 1: *The mases of the MSSM Higgs bosons as a function of* M_A *for two values* $\tan\beta = 3$ *and 30, in both the no mixing and maximal mixing scenarios. The full set of radiative corrections is included.*

For the mixing angle α of the CP–even Higgs sector, including only the dominant radiative correction given above, it is given in terms of $M_A, \tan\beta$ and ϵ, by

$$\tan 2\alpha = \tan 2\beta \, \frac{M_A^2 + M_Z^2}{M_A^2 - M_Z^2 + \epsilon/\cos 2\beta} \ , \qquad -\frac{\pi}{2} \leq \alpha \leq 0 \ . \tag{2.87}$$

Here again, the accuracy of the formula, compared to the case where the full corrections are taken into account, is rather good.

We turn now to the couplings of the Higgs bosons, which determine to a large extent, the production cross sections and the decay widths. The pseudoscalar Higgs boson has couplings to isospin down (up) type fermions that are (inversely) proportional to $\tan\beta$ and, because of CP–invariance, it has no tree–level couplings to gauge bosons. In the case of the CP–even h and H bosons, the mixing angle α enters in addition. The couplings of the CP–even Higgs bosons to down (up) type fermions are enhanced (suppressed) compared to the SM Higgs couplings for values $\tan\beta > 1$; the couplings to gauge bosons are suppressed by $\sin(\beta-\alpha)$ or $\cos(\beta-\alpha)$ factors; see Table 1. In fact, the h and H bosons share the couplings of the SM Higgs to the gauge bosons; also, the squared sum of the couplings to fermions do not depend on α and obey the sum rules:

$$g_{hdd}^2 + g_{Hdd}^2 = 1/\cos^2\beta \ , \ g_{huu}^2 + g_{Huu}^2 = 1/\sin^2\beta \ , \ g_{hVV}^2 + g_{HVV}^2 = 1 \tag{2.88}$$

The couplings of the neutral Higgs bosons are displayed in Table 1 together with their limiting values in the decoupling regime, i.e. for $M_A \gg M_Z$. One can see that in the decoupling limit the CP–even h boson has SM–like Higgs couplings to gauge bosons and fermions. The heavier H boson with a mass approximately equal to M_A in this regime, has couplings of the same order as the A boson, i.e. it has rather small couplings to gauge bosons and couples to up-type and down–type fermions proportionally to, respectively, $1/\tan\beta$ and $\tan\beta$. The couplings, normalized to the SM Higgs values, are shown in Fig. 2 for $\tan\beta = 3$ and 30 for the maximal mixing scenario.

Φ	$g_{\Phi\bar{u}u}$	$g_{\Phi\bar{d}d}$	$g_{\Phi VV}$
h	$\cos\alpha/\sin\beta \to 1$	$-\sin\alpha/\cos\beta \to 1$	$\sin(\beta-\alpha) \to 1$
H	$\sin\alpha/\sin\beta \to -1/\tan\beta$	$\cos\alpha/\cos\beta \to \tan\beta$	$\cos(\beta-\alpha) \to -1/\tan\beta$
A	$1/\tan\beta$	$\tan\beta$	0

Table 1: *Neutral Higgs boson couplings to fermions and gauge bosons in the MSSM normalized to the SM Higgs boson couplings, and their limits for $M_A \gg M_Z$.*

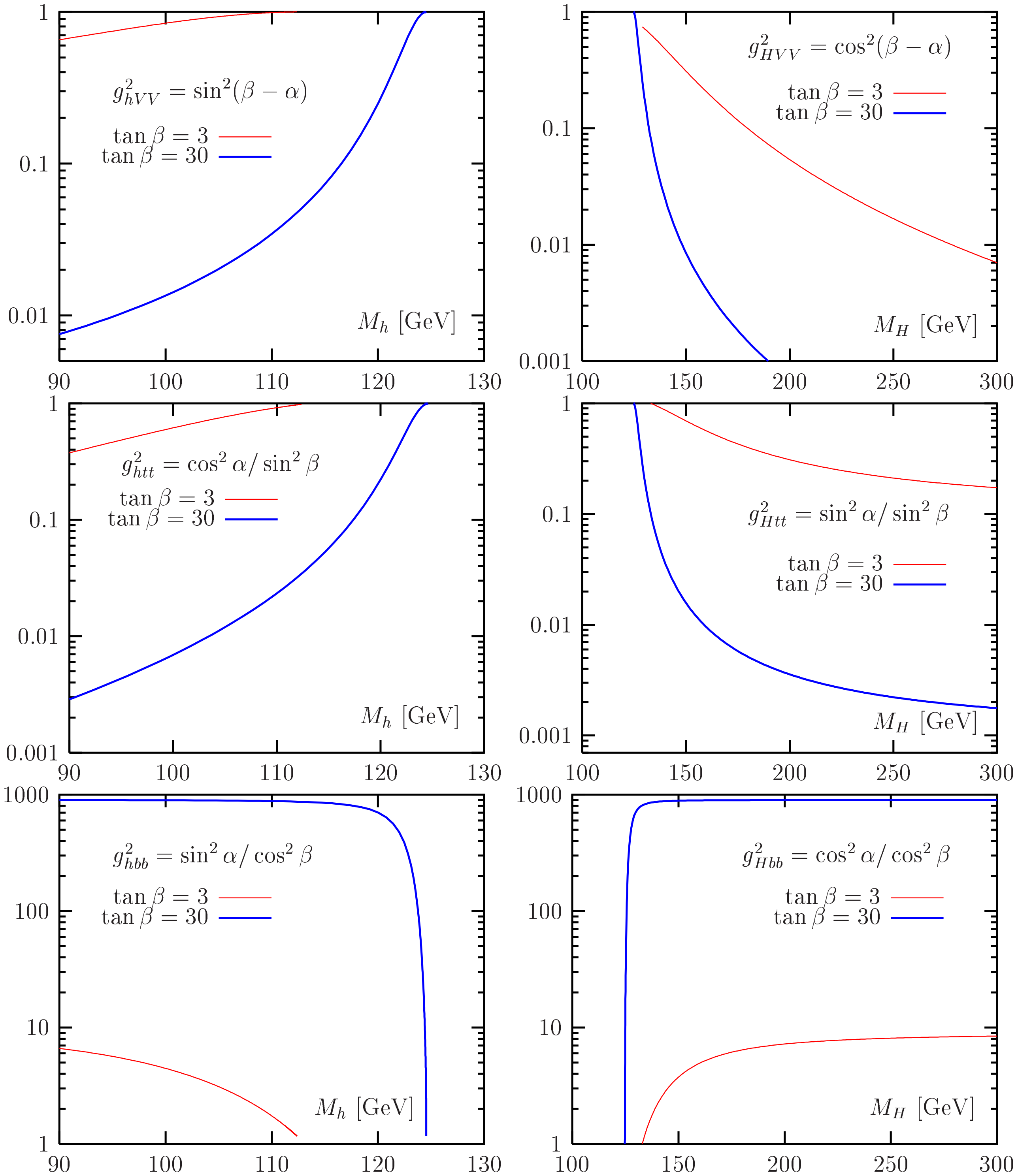


Figure 2: *The normalized couplings of the MSSM Higgs bosons to gauge bosons as a function of M_A for two values $\tan\beta = 3$ and 30, in the maximal mixing scenarios. The full set of radiative corrections is included.*

3. Production and decay properties of the Higgs bosons

In this section we will discuss the main decay modes of the Higgs bosons and their production cross sections in e^+e^- collisions, for masses relevant at LEP2 energies. That is, we will concentrate on the case of the lightest CP–even h boson and the pseudoscalar Higgs boson which can have masses below 100 GeV or so. We will then summarize the main constraints on the MSSM Higgs sector. Before that, we briefly summarize the rules which allow to calculate decay and production processes.

3.1 Rules for cross sections and decay widths

The differential cross section for a general $2 \to n$ process $i_1 i_2 \to f_1 \cdots f_n$ is given by

$$\mathrm{d}\sigma = \frac{|M(i_1 i_2 \to f_1..f_n)|^2}{4[(p_1 \cdot p_2)^2 - m_1^2 m_2^2]^{1/2}} \left(\Pi_n \frac{d^3 p_f}{(2\pi)^3 2 e_f} \right) (2\pi)^4 \delta^4(\Sigma p_i - \Sigma p_f)\, S \tag{3.1}$$

where p_1 and p_2 are the four–momenta of the initial particles. The various terms can be evaluated in the following way:

• In the amplitude squared $|M|^2$, one has to average (sum) on the degrees of freedom (polarization, color factor) of the initial (final) particles.

• There is a symmetry factor $S = 1/n!$ for n identical initial or final particles.

• The flux factor in the denominator of the first term is $2(p_1+p_2)^2 = 2s$ for a $2 \to n$ process if the initial particles have zero–mass, $m_1 = m_2 = 0$. It is $2M$ for the decay of a particle with a mass M, i.e for a $1 \to n$ process.

• The phase–space for a two–body process $a + b \to f_1 + f_2$ is given by

$$\mathrm{dPS2} = \frac{1}{16\pi^2} \frac{d^3 p_1}{e_1} \frac{d^3 p_2}{e_2}\, \delta^4(p_a + p_b - p_1 - p_2) \tag{3.2}$$

One obtains

$$\begin{aligned}
&\int \frac{d^3 p_2}{e_2} \delta^4(p_a + p_b - p_1 - p_2) = \frac{1}{e_2} \delta(e_a + e_b - e_1 - e_2) \\
&\text{with :}\ \ |\vec{p}_2| = |\vec{p}_a + \vec{p}_b - \vec{p}_2| \text{ and } e_2^2 = |\vec{p}_2|^2 + m_2^2 \\
&\text{and}\ \ \mathrm{d}^3 p_1 = \mathrm{d}\Omega\, |p_1|^2\, \mathrm{d}|p_1| \text{ with } e_1^2 = |\vec{p}_1|^2 + m_1^2
\end{aligned} \tag{3.3}$$

In the c.m. frame, $w = e_a + e_b$, $w' = e_1 + e_2 = (m_2^2 + p^2)^{\frac{1}{2}} (m_1^2 + p^2)^{\frac{1}{2}}$, one has:

$$\begin{aligned}
&\frac{\mathrm{d}w'}{\mathrm{d}p} = p \left(\frac{1}{e_1} + \frac{1}{e_2} \right) \Rightarrow \mathrm{d}w' = p \mathrm{d}p \left(\frac{1}{e_1} + \frac{1}{e_2} \right) = e_1 \mathrm{d}e_1 \frac{e_1 + e_2}{e_1 e_2} \\
&= \frac{\mathrm{d}\Omega}{16\pi^2} |p| \frac{e_1 \mathrm{d}e_1}{e_1 e_2} \delta(w - w') = \frac{\mathrm{d}\Omega}{16\pi^2} |p| \frac{\mathrm{d}w'}{w'} \delta(w - w') \Rightarrow \frac{\mathrm{d}\Omega}{16\pi^2} \frac{|p|}{\sqrt{s}}
\end{aligned} \tag{3.4}$$

Where for the last equality, the integral over dw' has been performed. The differential cross section is then given by:

$$\frac{d\sigma}{d\Omega} = \frac{1}{2s} \times \Sigma|M(i_1 i_2 \to f_1 f_2)|^2 \times \frac{1}{16\pi^2}\left(\frac{|p|}{\sqrt{s}}\right) \times S \tag{3.5}$$

Note that the the momentum $|p|$ can be expressed in term of the phase space function λ

$$|p| = \frac{\sqrt{s}}{2}\lambda = \frac{\sqrt{s}}{2}[1 - m_1^2/s - m_2^2/s)^2 - 4m_1^2 m_2^2/s^2]^{\frac{1}{2}} \tag{3.6}$$

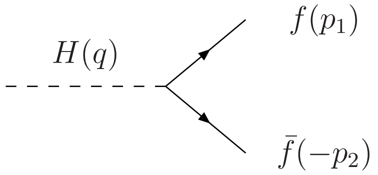
In the case of equal masses $m_1 = m_2$, it is given in terms of the velocity β:

$$|p| = \frac{\sqrt{s}}{2}\beta = \frac{\sqrt{s}}{2}(1 - 4m^2/s)^{1/2} \tag{3.7}$$

3.2 Main decay modes of the neutral Higgs bosons

3.2.1 Decays into fermions

In the SM, the amplitude and its conjugate for the decay of a Higgs boson into a pair of fermions, using the rules discussed in section 1.1, is given by:



$$-iM = \bar{u}^{s_1}(p_1)(im_f/v)v^{s_2}(-p_2)$$

$$+iM^\dagger = \bar{v}^{s_2}(-p_2)(-im_f/v)u^{s_1}(p_1)$$

The amplitude squared is then given by [there is no polarization for the Higgs]:

$$\sum_{s_1,s_2} MM^\dagger \;=\; N_c\left(\frac{m_f}{v}\right)^2 \sum_{s_1,s_2} \bar{v}^{s_2}(-p_2)\, u^{s_1}(p_1)\bar{u}^{s_1}(p_1)v^{s_2}(-p_2) \tag{3.8}$$

with $N_c = 3(1)$ for quarks (leptons). The amplitude squared is evaluated as:

$$\begin{aligned}
(v/m_f)^2/N_c \times \Sigma|M|^2 &= \mathrm{Tr}(\not{p}_1 + m_f)(-\not{p}_2 - m_f) \\
&= \mathrm{Tr}(\gamma_\mu p_1^\mu + m_f)(-\gamma_\nu p_2^\nu - m_f) \\
&= -p_1^\mu p_2^\nu \mathrm{Tr}(\gamma_\mu\gamma_\nu) - m_f^2 \mathrm{Tr}(1) \\
&= -4p_1.p_2 - 4m_f^2
\end{aligned} \tag{3.9}$$

Using: $q^2 = (p_1 - p_2)^2 = 2m_f^2 - 2p_1 \cdot p_2 = M_H^2$ and defining the velocity of the final fermions as previously discussed, $\beta_f = 2|p_f|/M_H = (1 - 4m_f^2/M_H^2)^{1/2}$, one obtains:

$$\Rightarrow \Sigma|M|^2 = N_c\,(m_f/v)^2\, 2(M_H^2 - 4m_f^2) = 2N_c\,(m_f^2/v^2)\, M_H^2\beta_f^2$$

The differential decay width is then simply given by:

$$\frac{\mathrm{d}\Gamma}{\mathrm{d}\Omega} = \frac{1}{2M_H} \times \Sigma|M|^2 \times \frac{1}{32\pi^2} \times \frac{2|p_f|}{M_H}$$

Integrating over $\mathrm{d}\Omega = \mathrm{d}\phi \mathrm{d}\cos\theta$ (and since there is no angular dependence, $\int \mathrm{d}\Omega = 4\pi$), one obtains the partial decay width:

$$\Gamma(H \to f\bar{f}) = N_c \frac{m_f^2}{v^2} \frac{M_H}{8\pi} \beta_f^3 \tag{3.10}$$

The SM Higgs boson decays dominantly into the heaviest fermion that is kinematically accessible and the decay width is proportional to M_H.

In the MSSM, the partial decay width of the CP–even Higgs bosons $\Phi = h, H$ are the same as in the SM case, except that they have to be multiplied by the reduced coupling squared, $g^2_{\Phi ff}$ given in Table 1,

$$\Gamma(\Phi \to f\bar{f}) = g^2_{\Phi ff} N_c \frac{m_f^2}{v^2} \frac{M_H}{8\pi} \beta_f^3 \tag{3.11}$$

In the case of the pseudoscalar Higgs boson, the situation is different. Indeed, the pseudoscalar nature of the coupling, proportional to $i\gamma_5$, will lead to a different form of the matrix element squared, which in this case is given by:

$$\begin{aligned}\sum_{s_1,s_2} MM^\dagger &\propto \sum_{s_1,s_2} \bar{v}^{s_2}(-p_2)\,(-\gamma_5)\,u^{s_1}(p_1)\bar{u}^{s_1}(p_1)\,(\gamma_5)\,v^{s_2}(-p_2) \\ &= \mathrm{Tr}(\not{p}_1 + m_f)(\gamma_5)(-\not{p}_2 - m_f)(-\gamma_5) = \mathrm{Tr}(\gamma_\mu p_1^\mu + m_f)(-\gamma_\nu p_2^\nu + m_f) \\ &= -4p_1.p_2 + 4m_f^2 = 2M_A^2 \end{aligned} \tag{3.12}$$

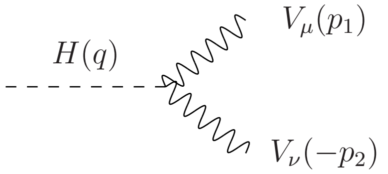
The difference between the CP–even and CP–odd cases is the therefore simply the absence of the β_f^2 term in the later case and the partial width is given by:

$$\Gamma(A \to f\bar{f}) = g^2_{Aff} N_c \frac{m_f^2}{v^2} \frac{M_A}{8\pi} \beta_f \tag{3.13}$$

Note that for Higgs decays into quarks, the QCD corrections turn out to be very large and should be included. The bulk of these corrections is taken into account when the pole quark masses $m_b \simeq 5$ GeV and $m_c \simeq 1.6$ GeV, are replaced by the running quark masses at the scale M_H in the $\overline{\mathrm{MS}}$ scheme, $\bar{m}_b \simeq 3$ GeV and $\bar{m}_c = 0.6$ GeV. The tree–level decay widths into bottom and charm quarks are then reduced by a factor 2 and 3, respectively.

3.2.2 Decays into massive gauge bosons

Again for a SM Higgs boson, the amplitude for the decay width into two real massive gauge bosons is given by:



$$-iM = \epsilon^*_\mu(p_1)\,(iM_V^2/v\, g^{\mu\nu})\,\epsilon^*_\nu(-p_2)$$

$$+iM^\dagger = \epsilon_{\mu'}(p_1)\,(-iM_V^2/v\, g^{\mu'\nu'})\,\epsilon_{\nu'}(-p_2)$$

$$\begin{aligned}
\sum_{\rm pol}|M|^2 &= \frac{M_V^4}{v^2}\, g^{\mu\nu}\, g^{\mu'\nu'} \sum_{\rm pol}\epsilon^*_\mu(p_1)\epsilon_{\mu'}(p_1) \sum_{\rm pol}\epsilon^*_\nu(-p_2)\epsilon_{\nu'}(-p_2) \\
(v^2/M_V^4)\Sigma &= g^{\mu\nu}\, g^{\mu'\nu'}\,(g_{\mu\mu'} - p_{1\mu}p_{1\mu'}/M_V^2)(g_{\nu\nu'} - p_{2\nu}p_{2\nu'}/M_V^2) \\
&= (g_{\mu\mu'} - p_{1\mu}p_{1\mu'}/M_V^2)(g^{\mu\mu'} - p_2^\mu p_2^{\mu'}/M_V^2) \\
&= 4 - p_1^2/M_V^2 - p_2^2/M_V^2 + (p_1\cdot p_2)^2/M_V^4 \\
&= (M_H^4/4M_V^4)\,[1 - 2M_V^2/M_H^2 + 12M_V^4/M_H^4]
\end{aligned}$$

The differential decay width is given by the usual formula,

$$\frac{{\rm d}\Gamma}{{\rm d}\Omega} = \frac{1}{2M_H} \times |M|^2 \times \frac{1}{4\pi}\,\frac{1}{8\pi} \times \frac{2|p_V|}{M_H} \times S \tag{3.14}$$

with a factor $S = \delta_V = \frac{1}{2}$ for two identical final Z bosons. This finally gives:

$$\Gamma(H\to VV) = \frac{\delta_V M_H^3}{32\pi v^2}\left(1 - \frac{4M_V^2}{M_H^2}\right)^{1/2}\left(1 - 2\frac{M_V^2}{M_H^2} + 12\frac{M_V^4}{M_H^4}\right) \tag{3.15}$$

The dependence on the gauge boson mass M_V is hidden, since $v \equiv 2M_W/g_2 = 2M_Z c_W/g_2$. For large enough M_H values [recall that $H\to f\bar{f} \propto M_H$], one has:

$$\Gamma(H\to VV) \simeq \delta_V M_H^3/(32\pi v^2) \Rightarrow \Gamma(H\to WW) \simeq 2\Gamma(H\to ZZ)$$

And the decay widths grows like M_H^3 i.e. is very large for $M_H \gg M_W$. For small M_H, one (two) gauge bosons can be off–shell, and the partial decay width is then given by

$$\begin{aligned}
\Gamma &= \frac{\Gamma_0}{\pi^2}\int_0^{M_H^2}\frac{{\rm d}q_1^2 M_V\Gamma_V}{(q_1^2 - M_V^2)^2 + M_V^2\Gamma_V^2}\int_0^{M_H^2 - q_1^2}\frac{{\rm d}q_2^2 M_V\Gamma_V}{(q_2^2 - M_V^2)^2 + M_V^2\Gamma_V^2} \\
\Gamma_0 &= \frac{\delta_V M_H^3}{32\pi v^2}\,\lambda^{1/2}\left(\lambda - \frac{12q_1^2q_2^2}{M_H^4}\right)\ ,\ \lambda = \left(1 - \frac{q_1^2}{M_H^2} - \frac{q_1^2}{M_H^2}\right)^2 - \frac{4q_1^2q_2^2}{M_H^4}
\end{aligned}$$

In the MSSM, the decay widths of the CP–even Higgs bosons can be obtained from the previous expressions by multiplying with the reduced couplings to Z and W bosons,

$$\Gamma(\Phi\to VV) = g^2_{\Phi VV}\Gamma(H_{\rm SM}\to VV) \tag{3.16}$$

The CP–even A boson does not decay into VV pairs because of CP–invariance.

3.2.3 Decays into gluons, photons and photon + Z boson

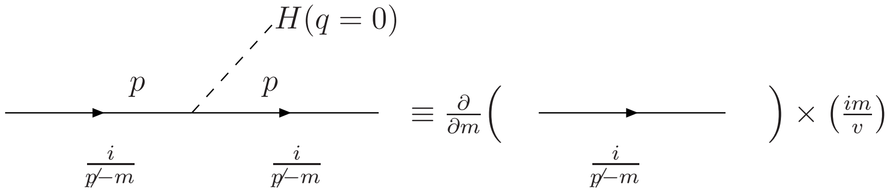
Higgs bosons do not couple to massless particles at tree-level; however couplings can be induced by loops of heavy particles. We have vertex diagrams with fermion (top only) and W boson exchange for $H \to \gamma\gamma, Z\gamma$ and only top loops for $H \to gg$. The calculation is rather complicated. However it becomes simple if the Higgs boson momentum is small, i.e. if the Higgs boson is much lighter than the loop particle, $M_H \ll M_{\rm loop}$. In this case, one can use a low–energy theorem which relates the amplitudes of two processes which differ only by the emission of a Higgs boson. Indeed, if one recalls the discussion in section 2.1, the coupling of a Higgs boson to a particle with mass m_i is generated by simply performing the substitution

$$m_i \to m_i(1 + H/v) \tag{3.17}$$

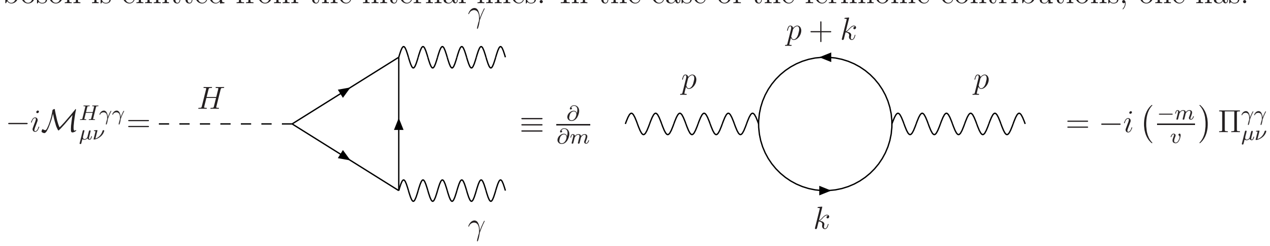
in the Lagrangian, where the Higgs boson is a constant field. This implies the following relation between two matrix elements with and without a Higgs field with zero–momentum q in the SM

$$\lim_{q\to 0} \mathcal{M}(X \to Y + H) = \frac{1}{v} m_i \frac{\partial}{\partial m_i} \mathcal{M}(X \to Y) \tag{3.18}$$

with m_i the masses of the particles involved in the process $X \to Y$. In the case of fermions, this can be viewed in a simple way by considering the relation between fermion propagator and the Higgs–fermion coupling:



Thus the $H \to \gamma\gamma$ amplitude can be related to the photon self–energy, where the Higgs boson is emitted from the internal lines. In the case of the fermionic contributions, one has:



So let us calculate the derivative of the fermionic photon-self energy. In the previous section, we arrived at the expression:

$$\Pi^{\gamma\gamma}_{\mu\nu}(p) = \frac{N_c e_f^2 e^2}{2\pi^2}(g_{\mu\nu}p^2 - p_\mu p_\nu)\int_0^1 {\rm d}x \int_0^\infty y dy \frac{2x(1-x)}{[y + m^2 - p^2 x(1-x)]^2} \tag{3.19}$$

We can now calculate the $H\gamma\gamma$ vertex [note that here, the photons are on–shell and $p_1 \neq p, p_2 \neq p$ but we have $p^2 = p_1 \cdot p_2 = M_H^2/2$]

$$\begin{aligned} \mathcal{M}_{\mu\nu}^{H\gamma\gamma} &= -\frac{m}{v}\frac{\partial}{\partial m}\Pi_{\mu\nu}^{\gamma\gamma}(p_1,p_2) = -\frac{2m^2}{v}\frac{\partial}{\partial m^2}\Pi_{\mu\nu}^{\gamma\gamma}(p_1,p_2) \\ &= -\frac{m^2}{v}\frac{N_c e_f^2 e^2}{\pi^2}(g_{\mu\nu}p_1.p_2 - p_{1\mu}p_{2\nu})\int_0^1 \mathrm{d}x \int_0^\infty \frac{-2x(1-x)ydy}{[y+m^2-p^2x(1-x)]^3} \end{aligned} \quad (3.20)$$

Inside the integral, we can suppose that $m^2 \gg p^2(M_H^2)$ and integrate over the variables x and y [$\int x(1-x)dx = \frac{1}{6}$ and $\int y(y+m^2)^{-3}dy = -\frac{1}{2m^2}$]

$$\mathcal{M}_{\mu\nu}^{H\gamma\gamma} \;=\; \frac{2}{3v}\,N_c\,e_f^2\,\frac{\alpha}{\pi}\,(g_{\mu\nu}p_1.p_2 - p_{1\mu}p_{2\nu})$$

Now we use the same machinery as for the decays into gauge bosons:

$$|\mathcal{M}|^2 = \frac{4}{9v^2}N_c^2 e_f^4 \frac{\alpha^2}{\pi^2}\,\frac{M_H^4}{4}\,\Sigma|g^{\mu\nu}\epsilon_\mu^*(p_1)\epsilon_\nu^*(p_2)|^2 = \frac{4M_H^4}{9v^2}N_c^2 e_f^4\frac{\alpha^2}{\pi^2} \quad (3.21)$$

Integrating over phase space (and with a factor 1/2 for identical photons), one obtains:

$$\Gamma(H\to\gamma\gamma) = \frac{M_H^3}{9v^2}N_c^2 e_f^4\frac{\alpha^2}{16\pi^3} \quad (3.22)$$

Several remarks are to be made at this stage:

– The amplitude was of course finite since there was no tree level contribution.

– The approximation $m_f \gg M_H$ is in practice good up to $M_H \sim 2m_f$.

– Only top quarks contribute here, the other fermions have negligible Yukawa coupling.

– Infinitely heavy fermions do not decouple from the amplitude; thus, this is a way to count the number of heavy particles coupling to the Higgs boson!

– This is only the contribution of one fermion species, there are also contributions from W bosons. In the limit, $M_H \ll M_W$, one would obtain (the calculation is more complicated and the limit not relevant for heavy Higgs bosons):

$$\Gamma(H\to\gamma\gamma) = \frac{M_H^3}{9v^2}\frac{\alpha^2}{16\pi^3}\left|\Sigma_f\, N_c e_f^2 - \frac{21}{4}\right|^2 \quad (3.23)$$

– The W contribution is larger, a factor of ~ 5, than the top quark contribution.

– With the same calculation, one can get the amplitude for the decay $H \to Z\gamma$. The only difference is in the Z couplings and the Z mass in the phase space. One obtains?:

$$\Gamma(H\to Z\gamma) = \frac{M_H^3}{9v^2}\frac{\alpha^2}{8\pi^3}\left(1-\frac{M_Z^2}{M_H^2}\right)^{1/2} \times 2 \times \left|\Sigma_f\, N_c\, e_f v_f - \cot\theta_W \frac{21}{4}\right|^2 \quad (3.24)$$

Here again, the W contribution is much larger, a factor of $\gtrsim 10$, than that of the top quark.

– The previous calculation holds also for gluons if we make the appropriate changes: $Q_e e \to g_s T_a$ which means $\alpha \to \alpha_s$ and $N_c^2 \to |\mathrm{Tr}(T_a T_a)|^2 = |\frac{1}{2}\delta_{ab}|^2 = 2$:

$$\Gamma(H \to gg) = \frac{M_H^3}{9v^2}\frac{\alpha_s^2}{8\pi^3} \tag{3.25}$$

– For the CP–even Higgs bosons in the MSSM, $\Phi = H, H$, the partial width are obtained from the previous expressions if the Higgs couplings to fermions and gauge bosons, $g_{\Phi ff}$ and $g_{\Phi VV}$ are included. However, for large $\tan\beta$ values, the Higgs couplings to bottom quarks can be strongly enhanced while the couplings to top quarks and gauge bosons are suppressed, and the b–quark loop becomes dominant. In this case, the low energy theorem cannot be used since the Higgs mass is larger than m_b. However, in this case, the branching ratios for the loop decays are in general very small.

– Finally, for the pseudoscalar Higgs boson, the loop decays involve only the fermion loops since the AVV couplings are absent. The results are different form the CP–even Higgs case because of the presence of the γ_5 couplings. In addition, for large $\tan\beta$, the b–quark loop contributions are also dominant and the low energy theorem cannot be used. But again in this case, the branching ratios are also very small.

3.2.4 Total decay widths and branching ratios

The decay pattern of the Higgs bosons of the MSSM is rather complicated and depends strongly on the value of $\tan\beta$; see Fig. 3. For the heavier neutral Higgs particles, besides the decays discussed previously, one has also several additional decays: decays of the CP–even Higgs boson into two light Higgs bosons $H \to hh$, and decays of the pseudoscalar Higgs particle into hZ final states, $A \to hZ$. Let us summarize the various modes.

The lightest h boson will decay mainly into fermion pairs since its mass is smaller than ~ 130 GeV. This is, in general, also the dominant decay mode of the pseudoscalar boson A. For values of $\tan\beta$ much larger than unity, the main decay modes of the three neutral Higgs bosons are decays into $b\bar{b}$ and $\tau^+\tau^-$ pairs with the branching ratios being of order $\sim 90\%$ and 10%, respectively. For large masses, the top decay channels $H, A \to t\bar{t}$ open up, yet for large $\tan\beta$ these modes remain suppressed. If the masses are high enough, the heavy H boson can decay into gauge bosons or light h boson pairs and the pseudoscalar A particle into hZ final states. However, these decays are strongly suppressed for $\tan\beta \gtrsim 3$–5 as it is suggested by the LEP2 constraints to be discussed later.

Note that for the h boson near its maximal value and for the H particle near its minimal mass value, the particles are SM–like with masses close to $\sim 130 - 140$ GeV. In this case, the h or H bosons decays into a large variety of channels. The main decay mode are by far the decay into $b\bar{b}$ and WW^* pairs followed by the decays into $c\bar{c}$ and $\tau^+\tau^-$ pairs with branching ratios of a few %. Also of significance, the top–loop mediated Higgs decay into gluons, which for M_H around 120 GeV occurs at the level of $\sim 5\%$. The top and W–loop mediated $\gamma\gamma$ and $Z\gamma$ decay modes are very rare the branching ratios being of $\mathcal{O}(10^{-3})$.

The charged Higgs particles decay into fermions pairs: mainly $t\bar{b}$ and $\tau\nu_\tau$ final states for $H^\pm$ masses, respectively, above and below the tb threshold. If allowed kinematically and for small values of $\tan\beta$, the $H^\pm$ bosons decay also into hW final states for $\tan\beta \lesssim 5$.

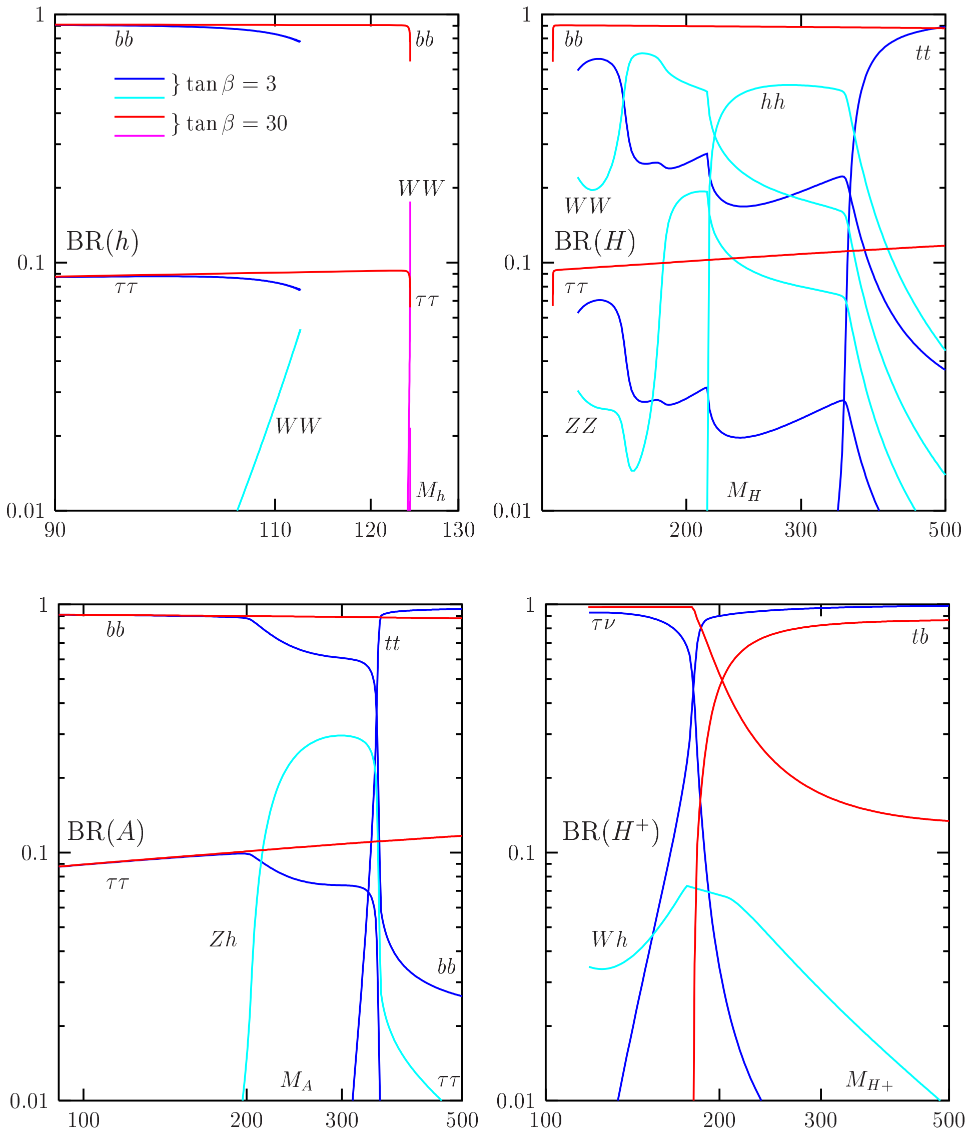


Figure 3: *Dominant MSSM Higgs bosons decay branching ratios as functions of the Higgs boson masses for* $\tan\beta = 3$ *and 30.*

Adding up the various decay modes, the widths of all five Higgs bosons remain very narrow. The total width of one the CP–even Higgs particle will be close to the SM Higgs width, while the total widths of the other Higgs particles will be proportional to $\tan\beta$ and will be of the order of 10 GeV even for large masses.

Other possible decay channels for the MSSM bosons, in particular the heavy H, A and $H^\pm$ states, are decays into supersymmetric particles. In addition to light sfermions, decays into charginos and neutralinos could eventually be important if not dominant. Decays of the lightest h boson into the lightest neutralinos (LSP) or sneutrinos can be also important, exceeding 50% in some parts of the SUSY parameter space. These decays can render the search for Higgs particle rather difficult, in particular at hadron colliders.

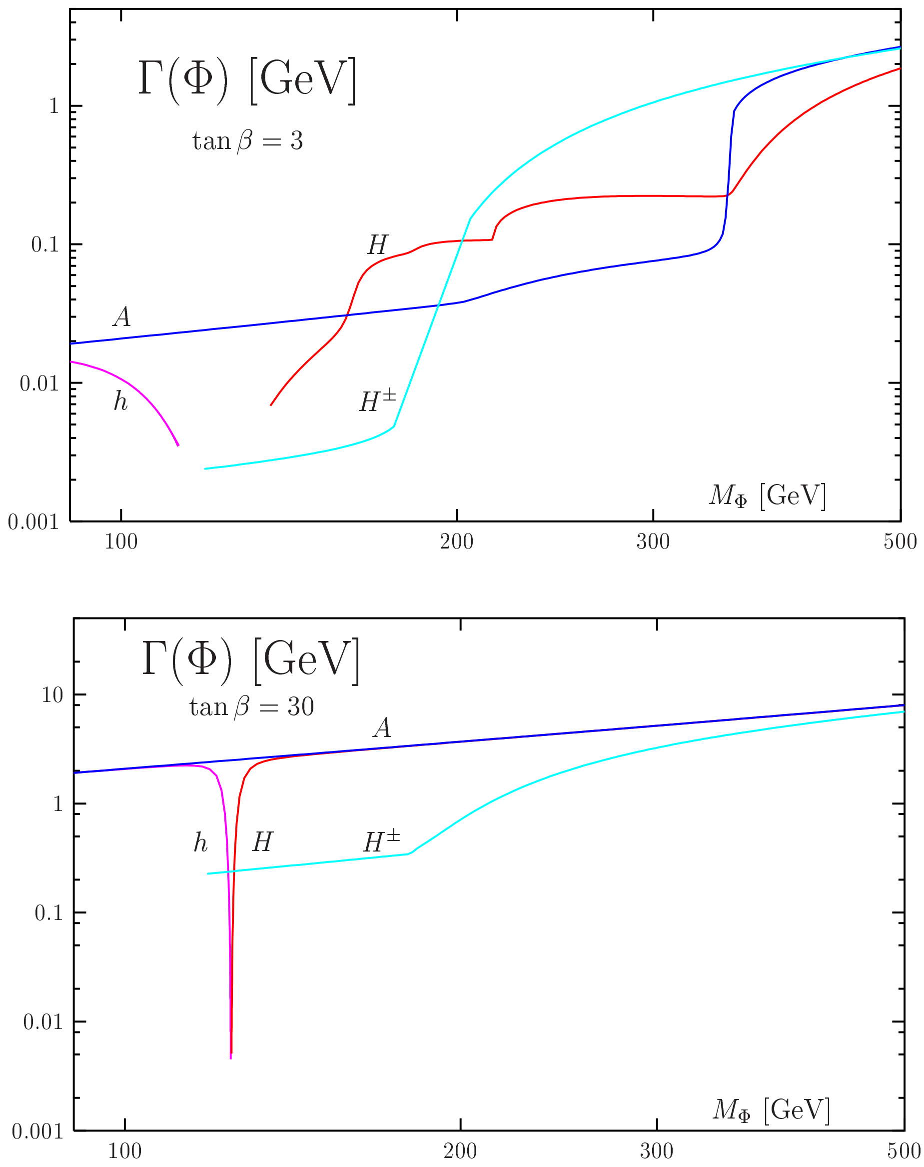
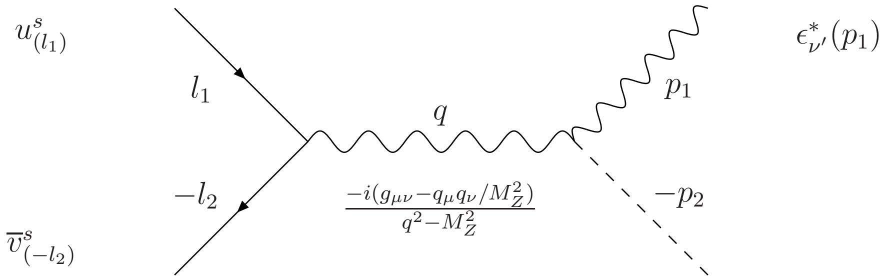


Figure 3 (continued): *The total decay widths of the MSSM Higgs bosons as functions of the Higgs boson masses for* $\tan\beta = 3$ *and 30.*

3.3 Production in e^+e^- collisions

3.3.1 The Higgs–strahlung process

Let us calculate the production cross section of the SM Higgs boson in the Higgs–strahlung process, $e^+e^- \to HZ$, which is the most important channel at LEP2. The Feynman diagram for the process is shown in the figure below, with the convention for the momenta of the particles involved, the spinors for the initial electrons, the polarization of the final Z boson and the propagator of the intermediate gauge boson.



Using the above ingredients, the amplitude for the process is written as

$$-iM = \overline{v}^s_{(-l_2)} ie\gamma_\mu(v_e - a_e\gamma_5)u^s_{(l_1)} \frac{-i(g^{\mu\nu} - q^\mu q^\nu/M_Z^2)}{q^2 - M_Z^2}\left(\frac{iM_Z^2}{v}g_{\nu\nu'}\right)\epsilon^*_{\nu'} \tag{3.26}$$

The first thing to do is to use for simplification the Dirac equation $\not l u(l) = m_e \simeq 0$:

$$\overline{v}^s_{(-l_2)}\gamma_\mu q^\mu u^s_{(l_1)} = \overline{v}^s_{(-l_2)}[-\not l_2 + \not l_1]u^s_{(l_1)} = 2m_e \sim 0 \Rightarrow q_\mu q_\nu \to 0 \tag{3.27}$$

where m_e is supposed to be much smaller than $\sqrt{s} = \sqrt{q^2}$. Then:

$$|M|^2 = \frac{e^2 M_Z^4 v^{-2}}{(q^2 - M_Z^2)^2}\epsilon^\nu_{(p_1)}\epsilon^\mu_{(p_1)}\overline{v}^s_{(-l_2)}\gamma_\mu(v_e - a_e\gamma_5)u^s_{(l_1)}\overline{u}^s_{(l_1)}\gamma_\nu(v_e - a_e\gamma_5)v^s_{(-l_2)} \tag{3.28}$$

and average over polarizations of the $e^\pm$ and sum on those of the photon:

$$\begin{aligned}
\frac{1}{4}\Sigma|M|^2 &= \frac{k}{4}\mathrm{Tr}\,\not l_1(v_e - a_e\gamma_5)\gamma_\mu(-\not l_2)(v_e - a_e\gamma_5)\gamma_\nu\left(-g^{\mu\nu} + \frac{p_1^\mu p_1^\nu}{M_Z^2}\right)\\
\mathrm{Tr} &= (v_e^2 + a_e^2)\mathrm{Tr}\,\not l_1\gamma_\mu\not l_2\gamma_\nu - 2a_e v_e \mathrm{Tr}\,\not l_1\gamma_\mu\not l_2\gamma_\nu\gamma_5\\
&= 4(v_e^2 + a_e^2)[l_{1\mu}l_{2\nu} + l_{2\mu}l_{1\nu} - l_1.l_2 g_{\mu\nu}] - 8ia_e v_e\, l_1^\alpha l_2^\beta\epsilon_{\alpha\mu\mu\beta\nu}\\
\frac{1}{4}\Sigma|M|^2 &= k(v_e^2 + a_e^2)\left[2(l_1.l_2) - 2\frac{(l_1.p_1)(l_2.p_1)}{M_Z^2} - 4(l_1.l_2) + (l_1.l_2)\frac{p_1^2}{M_Z^2}\right]\\
&= k(v_e^2 + a_e^2)\left[-(l_1.l_2) - 2(l_1.p_1)(l_2.p_1)/M_Z^2\right]
\end{aligned} \tag{3.29}$$

where we have used the fact that the tensor $(\epsilon_{\alpha\mu\beta\nu})\, g^{\mu\nu} - p_1^\mu p_1^\nu$ is (anti)symmetric.
In the e^+e^- c.m. frame, one has (with $E_{1,2}^2 = M_{Z,H}^2 + |p|^2$) and $|p| = \sqrt{s}/2\lambda$):

$$l_{1,2} = \frac{\sqrt{s}}{2}(\pm 1, 0, 0, 1) \text{ and } p_{1,2} = (\pm E_{Z,H}, 0, |p|\sin\theta, |p|\cos\theta) \tag{3.30}$$

$$\Rightarrow k(v_e^2+a_e^2)\left[\frac{s}{2}+\frac{s(E_Z^2-|p|^2\cos^2\theta)}{2M_Z^2}\right]=k(v_e^2+a_e^2)\frac{s^2}{M_Z^2}\left[\frac{M_Z^2}{s}+\frac{\lambda^2\sin^2\theta}{8}\right] \quad (3.31)$$

The differential cross section is then given by:

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\cos\theta\mathrm{d}\phi}=\frac{1}{2s}\left[\frac{e^2M_Z^4(v_e^2+a_e^2)s}{v^2(s-M_Z^2)^2M_Z^2}\left(\frac{M_Z^2}{s}+\frac{1}{8}\lambda^2\sin^2\theta\right)\right]\frac{\lambda}{32\pi^2} \quad (3.32)$$

with $\int \mathrm{d}\phi = 2\pi$ and $\int \sin^2\theta \mathrm{d}\cos\theta = 4/3$, one gets the cross section

$$\sigma(e^+e^-\to HZ)=\frac{\alpha M_Z^2}{48v^2}\frac{v_e^2+a_e^2}{s(1-M_Z^2/s)^2}\lambda(\lambda^2+12M_Z^2/s) \quad (3.33)$$

3.3.2 Properties of the process and implications for LEP2

A few remarks on this process can be made:

– The cross section drops like $1/s$ at high–energies (typical of an s–channel process). The maximum is reached at $\sqrt{s}=M_Z+\sqrt{2}M_H$.

– In the MSSM, the production cross sections of the CP–even Higgs bosons $\Phi=h,H$ are suppressed by the Higgs couplings to gauge bosons $g_{\Phi VV}$ compared to the SM. The cross sections are then given by

$$\sigma(e^+e^-\to\Phi Z)=g_{\Phi VV}^2\,\sigma(e^+e^-\to H_{\rm SM}Z) \quad (3.34)$$

In the decoupling regime, where h has SM–like couplings, the cross section for $e^+e^-\to hZ$ is maximal and has the same magnitude as in the SM case.

– At high energies $s\gg M_Z^2$, one has a differential cross section

$$\frac{\mathrm{d}\sigma}{\sigma\mathrm{d}\cos\theta}\simeq\frac{3}{4}\sin^2\theta \ \text{ with } \ \sigma\simeq\frac{\alpha M_Z^2}{48v^2}\frac{v_e^2+a_e^2}{s(1-M_Z^2/s)^2}\lambda^3 \quad (3.35)$$

the behavior in $\sin^2\theta$ of the angular distribution and in λ^3 of the total cross section is typical for the production of two spin–zero particles (recall that at a high–energies, the Z boson is almost a Goldstone boson).

– From this remark, one can obtain the cross section for the associated production of a scalar CP–even Higgs boson Φ and a pseudoscalar Higgs boson A, $e^+e^-\to\Phi A$. One has simply to replace M_Z by M_A, the $\lambda(\lambda^2+12M_Z^2/s)$ term by λ^3 for two spin–zero particle production, and plug in the ΦAZ coupling $g_{\Phi AZ}$

$$\sigma(e^+e^-\to\Phi A)=g_{\Phi AV}^2\,\sigma(e^+e^-\to H_{\rm SM}Z)\times\frac{\lambda^3}{\lambda(\lambda^2+12M_Z^2/s)} \quad (3.36)$$

Note that $g_{AhZ}^2=\cos^2(\beta-\alpha)$ while $g_{hZZ}^2=\sin^2(\beta-\alpha)$. In the decoupling limit the cross section for hA vanishes, while it is large when M_A is small, when $g_{hAZ}^2\sim 1$. The processes

$e^+e^- \to hZ$ and $e^+e^- \to Ah$ are thus complementary. In fact the sum of the cross sections of the two processes is approximately equal to the cross section for the production of a SM Higgs boson with a mass equal to M_h.

– In fact, this discussion can be extended to the case of the heavier CP–even Higgs boson. The complementarity is doubled in this case: there is a complementarity between the two processes $e^+e^- \to HZ$ and $e^+e^- \to HA$ as in the case above, but there is also a complementarity between the production of h and H in the two processes since:

$$\sigma(e^+e^- \to hZ) \propto \sin^2(\beta-\alpha) \text{ and } \sigma(e^+e^- \to HZ) \propto \cos^2(\beta-\alpha)$$
$$\sigma(e^+e^- \to hA) \propto \cos^2(\beta-\alpha) \text{ and } \sigma(e^+e^- \to HA) \propto \sin^2(\beta-\alpha) \tag{3.37}$$

At the maximum LEP2 center of mass energy, $\sqrt{s}=209$ GeV, the cross sections for the production of the lightest h boson in association with a Z boson and in association with the A boson are shown in Fig. 4, as a function of M_h for two values of $\tan\beta$, $\tan\beta=3$ (thick lines) and $\tan\beta=30$ (thin lines). We have chosen the maximal mixing scenario in the stop sector. Note that for an integrated luminosity of $\int \mathcal{L} \sim 100$ pb^{-1} a cross section of 10 fb means that we have 1 event.

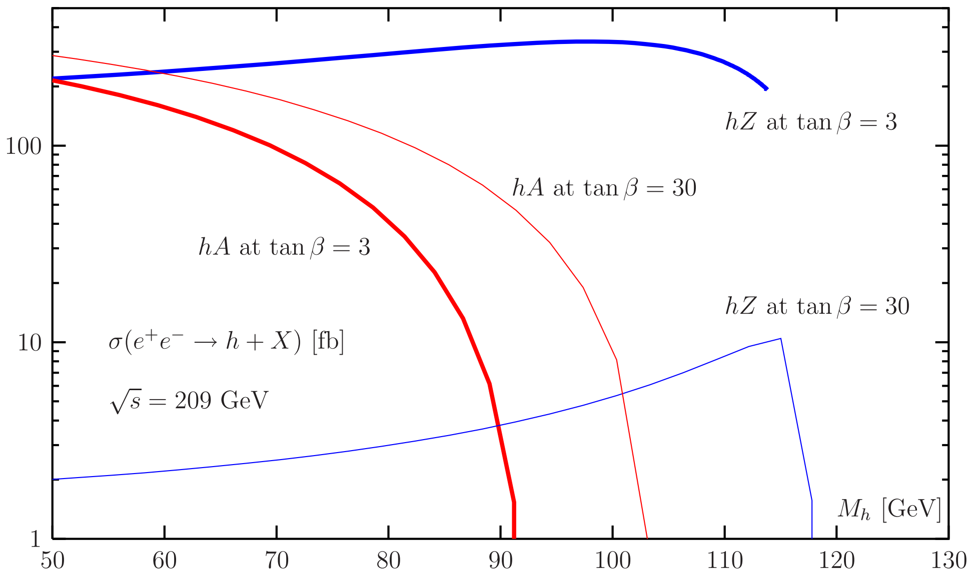


Figure 4: *The total cross section for h production at LEP2 with $\sqrt{s}=209$ GeV in the processes $e^+e^- \to hZ$ and $e^+e^- \to hA$ as a function of M_h for $\tan\beta=3$ and 30.*

For $\tan\beta=3$, only masses of the order of $M_h \sim 90$ GeV can be reached in the $e^+e^- \to hA$ process since the A is not degenerate with h and is heavier. However, for this small value of $\tan\beta$, the maximum value of M_h is about 113 GeV and therefore the h boson, which couples maximally to the Z boson is still kinematically accessible. The non–observation of h therefore rules out this $\tan\beta$ value in this scenario. For large $\tan\beta$ values, h and A are degenerate in mass for $M_h < M_h^{\rm max}$ and only the $e^+e^- \to hA$ process is at work, leading to a substantial cross section for M_A below 90 GeV. The $e^+e^- \to hZ$ process has always a cross section below 10 fb since the hZZ couplings is strongly suppressed [when it starts to be significant, the h boson becomes too heavy and the cross section is suppressed by phase space]. The constraints from LEP2 are discussed in more details in the next subsection.

3.4 Constraints on the MSSM Higgs sector

3.3.1 Constraints from LEP2

The search for the Higgs bosons was the main motivation for extending the LEP2 energy up to $\sqrt{s} \simeq 209$ GeV. In the SM, a lower bound $M_{H^0} > 114.1$ GeV has been set at the 95% confidence level, by investigating the Higgs–strahlung process, $e^+e^- \to ZH^0$. In the MSSM, this bound is valid for the lightest CP–even Higgs particle h if its coupling to the Z boson is SM–like, i.e. if $g^2_{ZZh} \equiv \sin^2(\beta-\alpha) \simeq 1$ [almost in the decoupling regime] or in the less likely case of the heavier H particle if $g^2_{ZZH} \equiv \cos^2(\beta-\alpha) \simeq 1$ [i.e. in the non–decoupling regime with a rather light M_H].

A complementary information is obtained from the search of Higgs bosons in the associated production processes $e^+e^- \to Ah$ where the 95% confidence level limits, $M_h > 91.0$ GeV and $M_A > 91.9$ GeV, on the h and A masses have been set. This bound is obtained in the limit where the coupling of the Z boson to hA pairs is maximal, $g^2_{ZhA}/g^2_{ZZH^0} \equiv \cos^2(\beta-\alpha) \simeq 1$, i.e. in the non–decoupling regime for large values of $\tan\beta$. This limit is lower than the one from the Higgs–strahlung process, due to the less distinctive signal and the β^3 suppression near threshold for spin–zero particle pair production. Note that for small M_A and large $\tan\beta$ values, M_H becomes small enough, Fig. 1, in the no–mixing scenario to allow for the possibility of the process $e^+e^- \to HA$, which is suppressed by $\sin^2(\beta-\alpha)$.

Deriving a precise bound on M_h for arbitrary values of M_A and $\tan\beta$ [i.e. not only in the decoupling limit or for $\tan\beta \gg 1$] and hence, for all values of the angle α, is more complicated since one has to combine results from two different production channels, which have different kinematical behavior, cross sections, backgrounds, etc.. However, some exclusion plots for $\sin^2(\beta-\alpha)$ versus M_h from the Higgs–strahlung process [and which can be used to constrain the mass of the H boson if $\sin^2(\beta-\alpha)$ is replaced by $\cos^2(\beta-\alpha)$] and $\cos^2(\beta-\alpha)$ versus $M_A + M_h$ [with $M_h \sim M_A$] from the pair production process, have been given in Refs. [6]. One can fit these exclusion contours and delineate the regions allowed by the LEP2 data up to $\sqrt{s} = 209$ GeV in the $[M_A, \tan\beta]$ and $[M_h, \tan\beta]$ planes in the no–mixing, typical mixing $A_t = M_S = 1$ Tev] and maximal mixing scenarii [the other SUSY parameters do not alter the picture]. The domains allowed by the LEP2 constraints are shown in Fig. 5.

In these figures the colored (blue, green and red) areas correspond to the domains of the

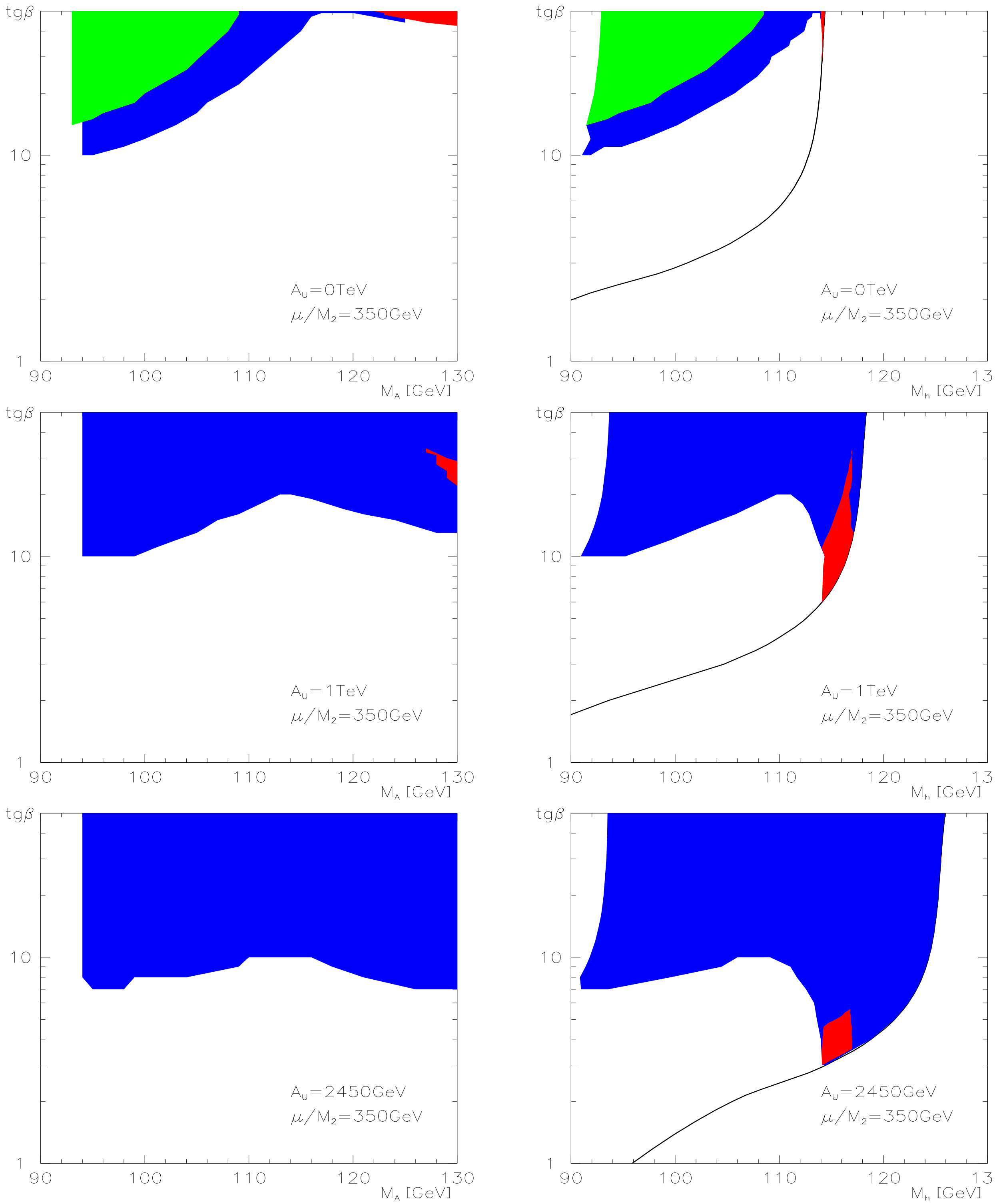


Figure 5: *The allowed regions for M_A [left] and M_h [right] from LEP2 searches as a function of* $\tan\beta$ *(colored regions, or black, dark gray and light gray regions) in the case of maximal (bottom), typical (central) and no–mixing (up). The red (or dark gray) regions indicate where* 114 *GeV*$<$ $M_h < 117$ *GeV and* $\sin^2(\beta-\alpha) > 0.9$ *and the green (or light gray) regions indicate where* 114 *GeV*$<$ $M_H < 117$ *GeV and* $\cos^2(\beta-\alpha) > 0.9$.

parameter space which are still allowed by the LEP2 searches. As can be seen, the allowed regions are different for the three scenarii of mixing in the stop sector.

In the no–mixing case, values $\tan\beta \lesssim 10$ are excluded for any value of the pseudoscalar mass $M_A \gtrsim 90$ GeV. For a light A boson, $M_A \lesssim 100$ GeV and $\tan\beta \gtrsim 10$, the masses $M_h \sim M_A$ pass the bound ($\gtrsim$ 91 GeV) from the Higgs pair production process, while H is light enough ($M_H \lesssim 115$ GeV) to be probed in the Higgs–strahlung process. For increasing M_A, this situation holds only for larger $\tan\beta$: for instance, for $M_A \gtrsim 115$ GeV, one needs values $\tan\beta \gtrsim 40$ to cope with the experimental bounds [i.e. to make H light enough to be produced]. In the typical mixing case, values $\tan\beta \lesssim 10$ are excluded for M_A close to 90 or 130 GeV. For intermediate M_A values, one needs larger $\tan\beta$ values to evade the constraint from the Higgs–strahlung process since the g_{hZZ} coupling is increasing while M_h is relatively small, leading to a sizable $\sigma(e^+e^- \to hZ)$. [Note that here, there is a strong interplay between the variation of $\sin^2(\beta-\alpha)$ and M_h in the phase space in $\sigma(e^+e^- \to hZ)$, which explains the bump around $M_A \sim 115$ GeV.] For large M_A, $\tan\beta$ values close to 5 are still allowed since M_h can be larger than 114 GeV. Finally, in the maximal mixing case, values $\tan\beta \gtrsim 7$ are excluded for 90 GeV $\lesssim M_A \lesssim 130$ GeV. However, for large M_A, only values $\tan\beta \gtrsim 3$ are excluded since here, the large stop mixing increases the maximal value of M_h to the level where it exceeds the discovery limit, $M_h \gtrsim 114$ GeV.

We turn now to the implications of the 2.1σ evidence for a SM–like Higgs boson with a mass $M_{\Phi_H} = 115.6$ GeV, seen by the LEP collaborations in the Higgs–strahlung process. In view of the theoretical and experimental uncertainties, this result will be interpreted as favoring the mass range 114 GeV $\lesssim M_{\Phi_H} \lesssim 117$ GeV. Furthermore, since this Higgs boson should be almost SM–like, we will impose the constraint $g^2_{ZZ\Phi_H} \geq 0.9$ so that the cross section for the Higgs–strahlung process, $\sigma(e^+e^- \to \Phi Z)$, is maximized. Of course, at the same time [and in particular in the case where the "observed" Higgs boson is H] the experimental constraints from the pair production process, $M_A, M_h \gtrsim 91$ GeV, should be taken into account.

The green areas show the regions where the "observed" Higgs boson is the heavier CP–even H particle. This occurs only for no mixing with values $\tan\beta \gtrsim 15$ and $M_A \lesssim 110$ GeV. In this case, $M_H \sim 115$ GeV and has SM–like couplings, while M_h is still larger than 91 GeV. For large stop mixing, the H boson is always heavier than 117 GeV and cannot be observed. The red areas correspond to the regions where the "observed" Higgs boson is in turn the lighter h particle, while M_H is heavier than 117 GeV. These regions are larger for the typical mixing scenario: in the no–mixing case, it is unlikely that the h boson mass exceeds 114 GeV except for very large $\tan\beta$, while in the maximal mixing scenario, the h boson is usually heavier than 117 GeV, except for very low $\tan\beta$.

3.3.2 Constraints from the Tevatron

Finally, let us briefly discuss the constraints on the MSSM Higgs sector from the Higgs boson searches at the Tevatron [a summary of Higgs physics and the constraints can be found in Ref. [7] for instance]. The first important constraint is coming from decays of the top quarks into b–quarks and charged Higgs bosons, $t \to bH^+$. For $\tan\beta$ values around unity [which are ruled out as discussed above] and for large values of $\tan\beta$, the $H^- t\bar{b}$ coupling, $g_{H^-tb} \propto m_t/\tan\beta + m_b \tan\beta$, is very strong. The branching ratio $\mathrm{BR}(t \to bH^+)$ becomes then comparable and even larger than the branching ratio of the standard mode, $t \to bW^+$, which allows the detection of top quarks at the Tevatron. A search for this top decay mode, with the H^- boson decaying into $\tau\nu_\tau$ final states, by the CDF and D0 Collaborations allows to place stringent limits on the value of $\tan\beta$ for charged Higgs boson masses below ~ 150 GeV: for $M_{H^\pm} \simeq 120$ GeV [i.e. $M_A \simeq 90$ GeV] one has $\tan\beta \lesssim 50$–60, while the bound becomes very weak for $M_{H^\pm} \simeq 150$ GeV [i.e $M_A \simeq 130$ GeV] where one has $\tan\beta \lesssim 100$ [i.e. beyond the limit where the b–quark Yukawa coupling is perturbative].

Furthermore, constraints on the MSSM Higgs sector at the Tevatron can be derived by considering the associated production of the pseudoscalar A boson with $b\bar{b}$ pairs, $pp \to q\bar{q}/gg \to b\bar{b}A$, with the A boson decaying into $\tau^+\tau^-$ final states. The cross section is proportional to $\tan^2\beta$ and can be very large for $\tan\beta \gg 1$. In addition, since in this regime, one of the CP–even Higgs bosons is always degenerate in mass with the A boson and has couplings to $b\bar{b}$ [and $\tau^+\tau^-$] pairs which are also proportional to $\tan\beta$, the rate for $b\bar{b}\tau^+\tau^-$ final states due to Higgs bosons in this type of process is multiplied by a factor 2. Since there is no evidence for non–SM contributions in this final state as analyzed by the CDF Collaboration, one can place bounds in the plane spanned by $\tan\beta$ and M_A. The upper bound is $\tan\beta \lesssim 80$ for $M_A \gtrsim 90$ GeV, and is weaker for higher values of M_A, $\tan\beta \lesssim 100$ for $M_A \sim 130$ GeV. Therefore, this constraint is less severe than the one due to top decays into charged Higgs bosons.

There are two other processes for MSSM Higgs boson production which could be relevant at the Tevatron as will be discussed later. The gluon–gluon fusion mechanism, $gg \to \Phi$ with $\Phi \equiv h, H$ or A, can have large cross sections in particular for large $\tan\beta$ values, but the backgrounds in the main decay channel $\Phi \to b\bar{b}$ are too large while for the cleaner $\gamma\gamma$ decays, $\sigma \times \mathrm{BR}(\Phi \to \gamma\gamma)$ is too small. The associated production of the CP–even Higgs bosons h or H with a W or Z boson, has cross sections which are too small, in particular if the gauge bosons are required to decay leptonically. These processes will therefore be useful only when an integrated luminosity of a few femtobarn will be accumulated, i.e. only at the Run II of the machine as will be discussed in the following section.

4. SM and MSSM Higgs bosons at future colliders

In this section we discuss the prospects for discovering the MSSM Higgs particle at the upgraded Tevatron, the LHC and a high–energy e^+e^- linear collider. The possibility of studying the properties of the Higgs particles will be then commented. We will first summarize the case of the SM Higgs boson and then point out the main differences in the case of the MSSM Higgs bosons. For detailed discussions see Refs. [5, 7–9].

4.1 Higgs Production at Hadron Colliders

The main production mechanisms of neutral Higgs bosons in the SM at hadron colliders are the following processes [5]

$$\begin{array}{clrcl} (a) & \text{gluon} - \text{gluon fusion} & gg & \rightarrow & H \\ (b) & WW/ZZ \text{ fusion} & VV & \rightarrow & H \\ (c) & \text{association with } W/Z & q\bar{q} & \rightarrow & V+H \\ (d) & \text{association with } Q\bar{Q} & gg, q\bar{q} & \rightarrow & Q\bar{Q}+H \end{array}$$

The cross sections are shown in Fig. 6 for the LHC with $\sqrt{s} = 14$ TeV and for the Tevatron with $\sqrt{s} = 2$ TeV as functions of the Higgs boson masses.

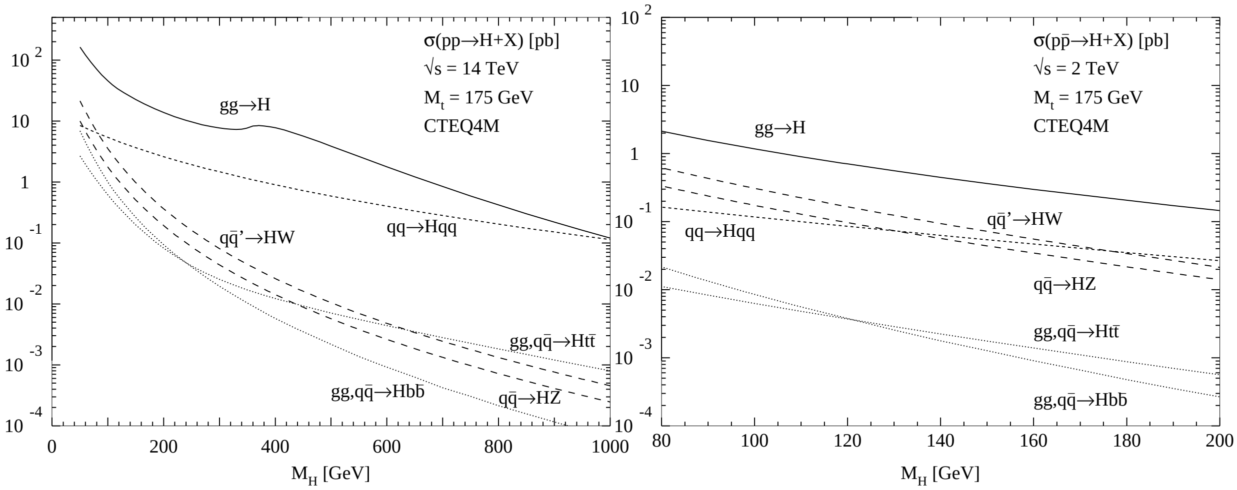


Figure 6: Higgs boson production cross sections at the LHC (left) and the Tevatron (right) for the various mechanisms as functions of the Higgs mass.

At the LHC, in the interesting mass range 100 GeV $\lesssim M_H \lesssim 250$ GeV, the dominant production process of the SM Higgs boson is by far the gluon–gluon fusion mechanism [in fact it is the case of the entire Higgs mass range] the cross section being of the order a few tens of pb. It is followed by the WW/ZZ fusion processes with a cross section of a few pb [which reaches the level of gg fusion for very large M_H]. The cross sections of the associated production with W/Z bosons or $t\bar{t}, b\bar{b}$ pairs are one to two orders of magnitude smaller than the gg cross section. Note that for an integrated luminosity $\int \mathcal{L} = (10)\ 100\ \text{fb}^{-1}$ in the low (high) luminosity option, $\sigma = 1$ pb would correspond to $10^4(10^5)$ events.

At the Tevatron, the most relevant production mechanism is the associated production with W/Z bosons, where the cross section is slightly less than a picobarn for $M_H \sim 120$ GeV, leading to $\sim 10^4$ Higgs events for a luminosity $\int \mathcal{L} = 20\ \text{fb}^{-1}$. The WW/ZZ fusion cross sections are slightly smaller for $M_H \lesssim 150$ GeV, while the cross sections for associated production with $t\bar{t}$ or $b\bar{b}$ pairs are rather low. The gg fusion mechanism has the largest cross section but suffers from a huge QCD two–jet background.

The next–to–leading order QCD corrections should be taken into account in the gg fusion processes where they are large, leading to an increase of the production cross sections by a factor of up to two. For the other processes, the QCD radiative corrections are relatively smaller: for the associated production with gauge bosons, the corrections [which can be inferred from the Drell–Yan W/Z production] are at the level of 10%, while in the case of the vector boson fusion processes, they are at the level of 30%. For the associated production with top quarks, the NLO corrections alter the cross section by $\sim 20\%$ if the scale is chosen properly. In all these production processes, the theoretical uncertainty, from the remaining scale dependence and from the choice of different sets of parton densities, can be estimated as being of the order of $\sim$ 20–30%.

The signals which are best suited to identify the produced Higgs particles at the Tevatron and at the LHC have been studied in great detail in Ref. [7, 8], respectively. I briefly summarize below the main conclusions of these studies.

At the Tevatron Run II, the associated production with W/Z bosons with the latter decaying leptonically lead to several distinct signatures in which a signal can be observed with sufficient integrated luminosity. In the low Higgs mass range, $M_H \lesssim 130$ GeV, the Higgs will mainly decay into $b\bar{b}$ pairs and the most sensitive signatures are $\ell\nu b\bar{b}$, $\nu\bar{\nu}b\bar{b}$, and $\ell^+\ell^- b\bar{b}$. Hadronic decays of the W and Z lead to the $q\bar{q}b\bar{b}$ final state and cannot be used since they suffers from large backgrounds from QCD multi-jet production. In the high Higgs mass range, $M_H \gtrsim 130$ GeV, the dominant decay is $H \to WW^*$ and the signature $\ell^\pm\ell^\pm jj$ from three vector boson final states can be used. In addition, one can use the final state $\ell^+\ell^-\nu\bar{\nu}$ with the Higgs boson produced in gg fusion.

The required luminosity to discover or exclude a SM Higgs boson, combining all channels in both D0 and CDF experiments, is shown in Fig. 7 as a function of M_H. With 15 fb^{-1} luminosity, a 5σ signal can be achieved for $M_H \lesssim 120$ GeV, while a Higgs boson with a mass $M_H \lesssim 190$ GeV can be excluded at the 95% confidence level.

Let us now turn to the signatures which can be used at the LHC. A discovery with a significance larger than 5σ can be obtained used various channels; see Figure 8.

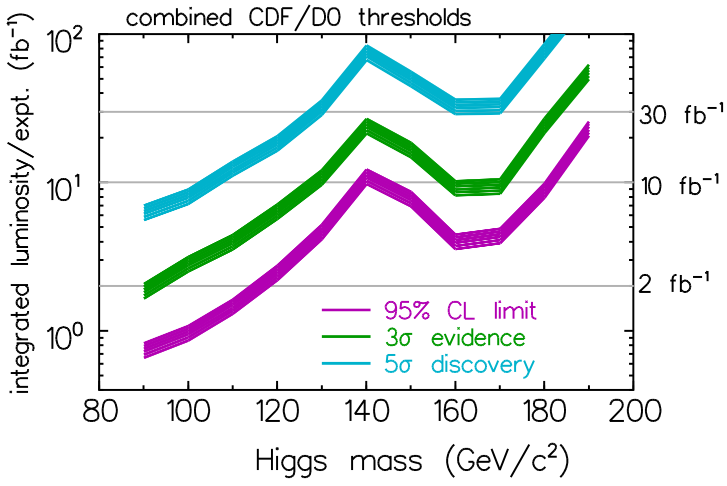


Figure 7: The integrated luminosity required per experiment, to either exclude a SM Higgs boson at 95% CL or discover it at the 3σ or 5σ level, as a function of M_H.

In the high mass range, $M_H \gtrsim 130$ GeV, the signal consists of the so–called "gold–plated" events $H \to ZZ^{(*)} \to 4\ell^{\pm}$ with $\ell = e, \mu$. The backgrounds, mostly $pp \to ZZ^{(*)}, Z\gamma^*$ for the irreducible background and $t\bar{t} \to WWb\bar{b}$ and $Zb\bar{b}$ for the reducible one, are relatively small. One can probe Higgs boson masses up to $\mathcal{O}(500$ GeV) with a luminosity $\int \mathcal{L} = 100$ fb^{-1}. The channels $H \to ZZ \to \nu\bar{\nu}\ell^+\ell^-$ and $H \to WW \to \nu\ell jj$, which have larger rates, allow to extend the reach to $M_H \sim 1$ TeV. The $H \to WW^{(*)} \to \nu\bar{\nu}\ell^+\ell^-$ [with H produced in gg fusion, and to a lesser extent, in association with W bosons] decay channel is very useful in the range 130 GeV $\lesssim M_H \lesssim 180$ GeV, where BR$(H \to ZZ^*)$ is too small, despite of the large background from WW and $t\bar{t}$ production.

For the "low mass" range, the situation is more complicated. The branching ratios for $H \to ZZ^*, WW^*$ are too small and due to the huge QCD jet background, the dominant mode $H \to b\bar{b}$ is practically useless. One has then to rely on the rare $\gamma\gamma$ decay mode with a branching ratio of $\mathcal{O}(10^{-3})$, where the Higgs boson is produced in the gg fusion and the associated WH and $Ht\bar{t}$ processes. A 5σ discovery can be obtained with a luminosity $\int \mathcal{L} = 100$ fb^{-1}, despite of the formidable backgrounds. Finally, in the very low mass range, $M_H \sim 115$ GeV, the channel $pp \to t\bar{t}H$ with $H \to b\bar{b}$ can be used.

In the MSSM, the production processes for the neutral CP–even Higgs particles are practically the same as for the SM Higgs. However, for large $\tan\beta$ values, one has to take the b quark [whose couplings are strongly enhanced] into account: its loop contributions in the gg fusion process [and also the extra contributions from squarks loops, which however decouple for high squark masses] and associated production with $b\bar{b}$ pairs. The cross sections for the associated production with $t\bar{t}$ pairs and W/Z bosons and the WW/ZZ fusion processes, are suppressed for at least one of the particles because of the coupling suppression. Because

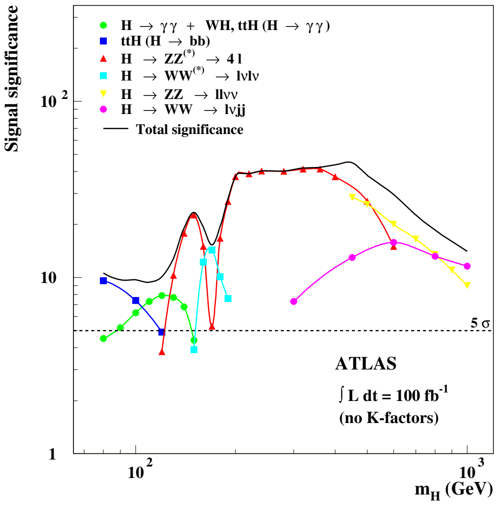


Figure 8: Significance for the SM Higgs boson discovery in various channels at the LHC with a high luminosity as a function of the Higgs mass.

of CP–invariance, the pseudoscalar A boson can be produced only in the gg fusion and in association with heavy quarks [associated production with a CP–even Higgs particle, $pp \to A + h/H$, is also possible but the cross section is too small]. For high enough $\tan\beta$ values and for $M_A \gtrsim (\lesssim)130$ GeV, the $gg/q\bar{q} \to b\bar{b} + A/H(h)$ and $gg \to A/H(h)$ processes become the dominant production mechanisms.

The charged Higgs particles, if lighter than the top quark, can be accessible in the decays $t \to H^+ b$ with $H^- \to \tau\nu_\tau$, leading to a surplus of τ events mimicking a breaking of τ versus e, μ universality. The $H^\pm$ particles can also be produced directly in the [properly combined] processes $gb \to tH^-$ or $qq/gg \to H^- t\bar{b}$.

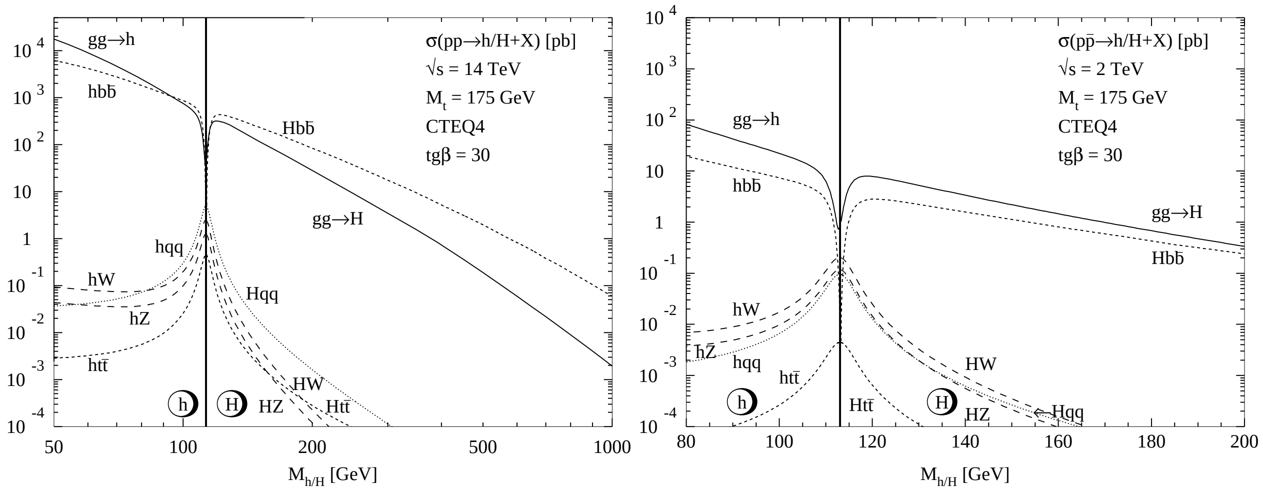


Figure 9: CP–even Higgs production cross sections at LHC (left) and Tevatron (right) for the various mechanisms as a function of the Higgs masses for $\tan\beta = 30$.

The cross sections for the production of the CP–even Higgs particles are shown in Fig. 9 for the Tevatron and LHC for $\tan\beta = 30$ [the cross sections for A production are roughly equal to the one of the $h(H)$ particle in the low (high) mass range]. The various detection signals can be briefly summarized as follows [see also Fig. 10]:

(i) Since the lightest Higgs boson mass is always smaller than ~ 130 GeV, the WW and ZZ signals cannot be used. Furthermore, the $hWW(h\bar{b}b)$ coupling is suppressed (enhanced) leading to a smaller $\gamma\gamma$ branching ratio than in the SM, making the search in this channel more difficult. If M_h is close to its maximum value, h has SM like couplings and the situation is similar to the SM case with $M_H \sim 100$–130 GeV.

(ii) Since A has no tree–level couplings to gauge bosons and since the couplings of H are strongly suppressed, the gold–plated ZZ signal is lost [for H it survives only for small $\tan\beta$ values, provided that $M_H < 2m_t$]. In addition, the $A/H \to \gamma\gamma$ signals cannot be used since the branching ratios are suppressed. One has then to rely on the $A/H \to \tau^+\tau^-$ or even $\mu^+\mu^-$ channels for large $\tan\beta$ values. [The decays $H \to hh \to b\bar{b}b\bar{b}$, $A \to hZ \to Zb\bar{b}$ and $H/A \to t\bar{t}$ have too small rates in view of the LEP2 constraints].

(iii) Light $H^\pm$ particles can be observed in the decays $t \to H^+b$ with $H^- \to \tau\nu_\tau$ where masses up to ~ 150 GeV can be probed. The mass reach can be extended up to a few hundred GeV for $\tan\beta \gg 1$, by considering the processes $gb \to tH^-$ and $gg \to t\bar{b}H^-$ with the decays $H^- \to \tau\nu_\tau$ [using τ polarization] or $\bar{t}b$.

(iv) All the previous discussion assumes that Higgs decays into SUSY particles are kinematically inaccessible. This seems to be unlikely since at least the decays of the heavier H, A and $H^\pm$ particles into charginos and neutralinos should be possible. Preliminary analyzes show that decays into neutralino/chargino final states $H/A \to \chi_2^0\chi_2^0 \to 4\ell^\pm X$ and $H^\pm \to \chi_2^0\chi_1^\pm \to 3\ell^\pm X$ can be detected in some cases. It could also be possible that the lighter h decays invisibly into the lightest neutralinos or sneutrinos. If this scenario is realized, the discovery of these Higgs particles will be more challenging. Preliminary analyzes for the 2001 les Houches Workshop in Ref. [8] show however, that an invisibly decaying Higgs boson could be detected in the WW fusion process.

(v) If top squarks are light enough, their contribution to the gg fusion mechanism and to the $\gamma\gamma$ decay [here, this is also the case for light charginos] should be taken into account. In the large mixing scenario, stops can be rather light and couple strongly to the h boson, leading to a possibly strong suppression of the product $\sigma(gg \to h) \times \mathrm{BR}(h \to \gamma\gamma)$. However, in this case, the associated production of the h boson with top squarks is possible and the cross sections would be rather sizable.

(vi) MSSM Higgs boson detection from the cascade decays of Supersymmetric particles, originating from squark and gluino production, are also possible. In particular, the production of the lighter h boson from the decays of the next-to-lightest neutralino and the production of $H^{\pm}$ from the decays of the heavier chargino/neutralino states into the lighter ones have been discussed.

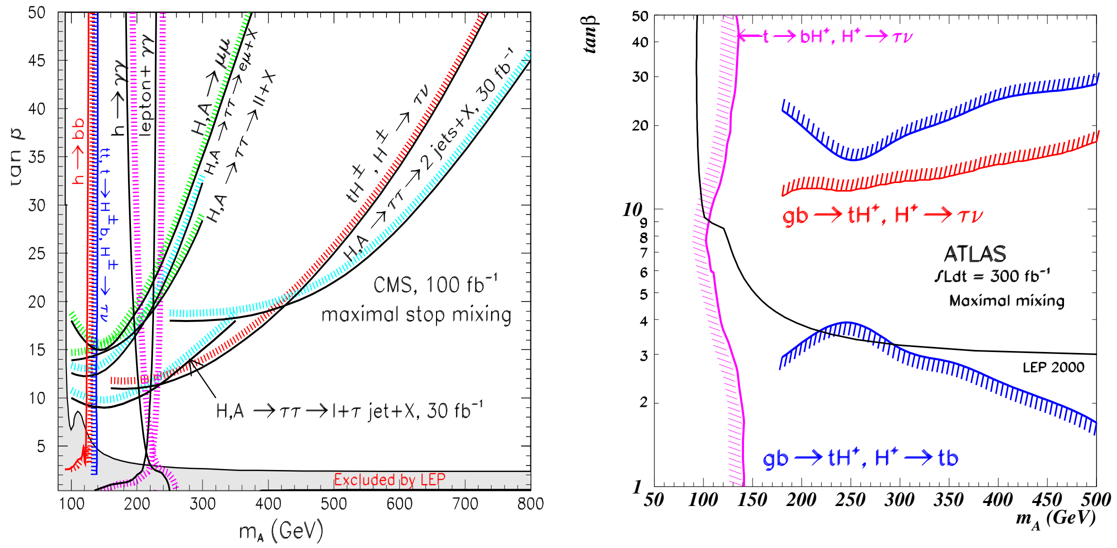


Figure 10: Neutral (left) and charged (right) MSSM Higgs boson discovery at the LHC in various channels in the $(M_A, \tan\beta)$ plane with a high luminosity.

At the Tevatron Run II, the search for the CP–even h and H bosons will be more difficult than in the SM because of the reduced couplings to gauge bosons, unless one of the Higgs particles is SM–like. However, associated production with $b\bar{b}$ pairs, $pp \to b\bar{b} + A/h(H)$ in the low (high) M_A range with the Higgs bosons decaying into $b\bar{b}$ pairs, might lead to a visible signal for rather large $\tan\beta$ values and M_A values below the 200 GeV range. The $H^{\pm}$ boson would be also accessible in top quark decays for large or small values of $\tan\beta$, for which the branching ratio BR$(t \to H^+ b)$ is large enough.

4.2 Higgs Production at e^+e^- Colliders

At e^+e^- linear colliders operating in the 300–800 GeV energy range, the main production mechanisms for SM–like Higgs particles are [5, 9]

$$
\begin{array}{llll}
(a) & \text{bremsstrahlung process} & e^+e^- & \to (Z) \to Z + H \\
(b) & WW \text{ fusion process} & e^+e^- & \to \bar{\nu}\ \nu\ (WW) \to \bar{\nu}\ \nu\ + H \\
(c) & ZZ \text{ fusion process} & e^+e^- & \to e^+e^-(ZZ) \to e^+e^- + H \\
(d) & \text{radiation off tops} & e^+e^- & \to (\gamma, Z) \to t\bar{t} + H
\end{array}
$$

The Higgs–strahlung cross section scales as $1/s$ and therefore dominates at low energies while the WW fusion mechanism has a cross section which rises like $\log(s/M_H^2)$ and dominates at high energies. At $\sqrt{s} \sim 500$ GeV, the two processes have approximately the same cross sections, $\mathcal{O}(100$ fb) for the interesting range 100 GeV $\lesssim M_H \lesssim$ 200 GeV, as shown in

Fig. 11. With an integrated luminosity $\int \mathcal{L} \sim 500$ fb^{-1}, as expected for instance at the TESLA machine [9], approximately 25.000 events per year can be collected in each channel for a Higgs boson with a mass $M_H \sim 150$ GeV. This sample is more than enough to discover the Higgs boson and to study its properties in detail.

The ZZ fusion mechanism has a cross section which is one order of magnitude smaller than WW fusion, a result of the smaller neutral couplings compared to charged current couplings. The associated production with top quarks has a very small cross section at $\sqrt{s} = 500$ GeV due to the phase space suppression but at $\sqrt{s} = 1$ TeV it can reach the level of a few femtobarn. Despite of the small production cross sections, shown in Fig. 11 as a function of $\sqrt{s}$ for $M_H = 120$ GeV, these processes will be very useful when it comes to study the Higgs properties as will be discussed later. The cross section for the double Higgs production in the strahlung process, $e^+e^- \to HHZ$, also shown in Fig. 11 is at the level of a fraction of a femtobarn and can be used to extract the Higgs self–coupling.

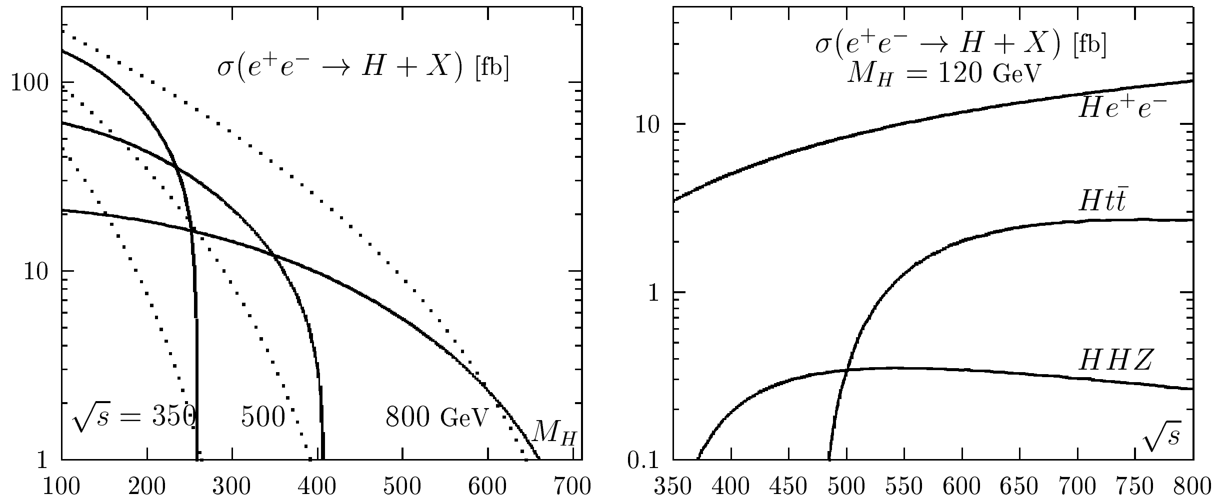


Figure 11: Production cross sections of the SM Higgs boson in e^+e^- in the main processes with $\sqrt{s} = 350, 500$ and 800 GeV as a function of M_H (left) and in higher order process as a function of $\sqrt{s}$ for $M_H = 120$ GeV (right).

In the Higgs–strahlung process, the recoiling Z boson [which can be tagged through its clean $\mu^+\mu^-$ decay] is mono–energetic and M_H can be derived from the energy of the Z if the initial e^+ and e^- beam energies are sharp [beamstrahlung, which smears out the c.m. energy should thus be suppressed as strongly as possible, and this is already the case for machine designs such as TESLA]. Therefore, it will be easy to separate the signal from the backgrounds. The WW fusion mechanism offers a complementary production channel.

It has been shown in detailed simulations that only a few fb^{-1} of integrated luminosity are needed to obtain a 5σ signal for a Higgs boson with a mass $M_H \lesssim 140$ GeV at a 500

GeV collider [in fact, in this case, it is better to go to lower energies where the cross section is larger], even if it decays invisibly [as it could happen in SUSY models for instance]. Higgs bosons with masses up to $M_H \sim 400$ GeV can be discovered at the 5σ level, in both the strahlung and fusion processes at an energy of 500 GeV and with a luminosity of 500 fb^{-1}. For even higher masses, one needs to increase the c.m. energy of the collider, and as a rule of thumb, Higgs masses up to $\sim 80\% \sqrt{s}$ can be probed. This means than a ~ 1 TeV collider will be needed to probe the entire SM Higgs mass range.

An even stronger case for e^+e^- colliders in the 300–800 GeV energy range is made by the MSSM. In e^+e^- collisions, besides the usual bremsstrahlung and fusion processes for h and H production, the neutral Higgs particles can also be produced pairwise: $e^+e^- \to A + h/H$. The cross sections for the bremsstrahlung and the pair production as well as the cross sections for the production of h and H are mutually complementary, coming either with a coefficient $\sin^2(\beta - \alpha)$ or $\cos^2(\beta - \alpha)$; see Fig. 12. The cross section for hZ production is large for large values of M_h, being of $\mathcal{O}(100$ fb) at $\sqrt{s} = 350$ GeV; by contrast, the cross section for HZ is large for light h [implying small M_H]. In major parts of the parameter space, the signals consist of a Z boson and $b\bar{b}$ or $\tau^+\tau^-$ pairs, which is easy to separate from the main background, $e^+e^- \to ZZ$ [in particular with b–tagging]. For the associated production, the situation is opposite: the cross section for Ah is large for light h whereas AH production is preferred in the complementary region. The signals consists mostly of four b quarks in the final state, requiring efficient $b\bar{b}$ quark tagging; mass constraints help to eliminate the QCD jets and ZZ backgrounds. The CP–even Higgs particles can also be searched for in the WW and ZZ fusion mechanisms.

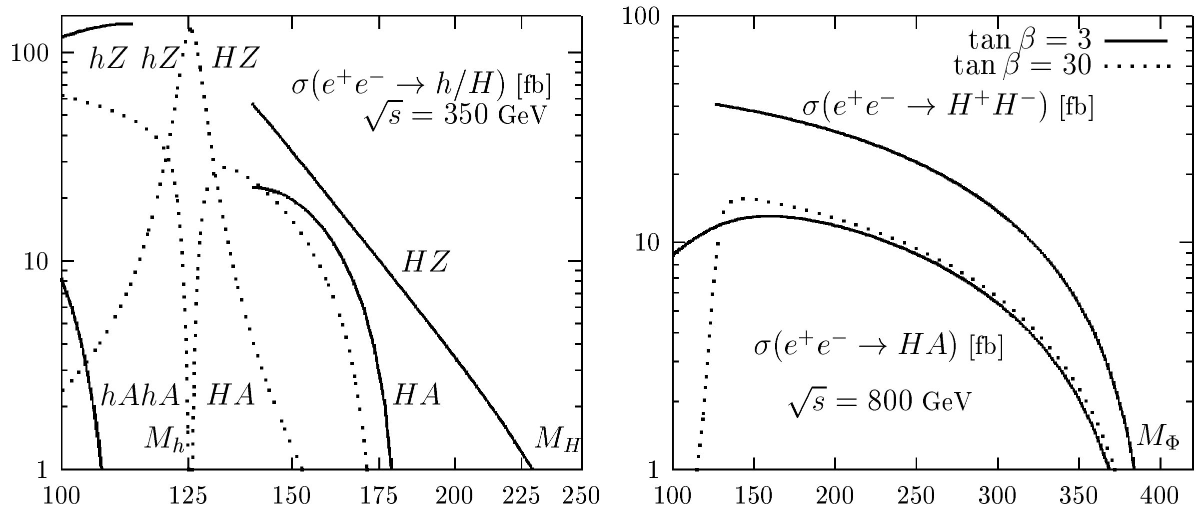


Figure 12: Production cross sections of the MSSM Higgs bosons in e^+e^- as functions of the masses: h, H production at $\sqrt{s} = 350$ GeV (left) and HA, H^+H^- production at $\sqrt{s} = 800$ GeV (right); the dotted (full) lines are for $\tan\beta = 30(3)$.

In e^+e^- collisions, charged Higgs bosons can be produced pairwise, $e^+e^- \to H^+H^-$, through γ, Z exchange. The cross section depends only on the charged Higgs mass; it is large almost up to $M_{H^\pm} \sim \sqrt{s}/2$. $H^\pm$ bosons can also be produced in top decays as at hadron colliders; in the range $1 < \tan\beta < m_t/m_b$, the $t \to H^+b$ branching ratio and the $t\bar{t}$ production cross sections are large enough to allow for their detection in this mode.

The discussion on the MSSM Higgs production at e^+e^- linear colliders [not mentioning yet the $\gamma\gamma$ option of the collider] can be summarized in the following points:

i) The Higgs boson h can be detected in the entire range of the MSSM parameter space, either through the bremsstrahlung process or pair production; in fact, this conclusion holds true even at a c.m. energy of 300 GeV and with a luminosity of a few fb^{-1}.

ii) All SUSY Higgs bosons can be discovered at an e^+e^- collider if the H, A and $H^\pm$ masses are less than the beam energy; for higher masses, one simply has to increase $\sqrt{s}$.

iii) Even if the decay modes of the Higgs bosons are very complicated [e.g. they decay invisibly] missing mass techniques allow for their detection.

iv) The additional associated production processes with $t\bar{t}$ and $b\bar{b}$ allow for the measurement of the Yukawa couplings. In particular, $e^+e^- \to b\bar{b} + H/A$ for high $\tan\beta$ values allows for the determination of this important parameter for low M_A values.

Finally, future linear colliders can be turned to $\gamma\gamma$ colliders, in which the photon beams are generated by Compton back–scattering of laser light; c.m. energies of the order of 80% of the e^+e^- collider energy and integrated luminosities $\int \mathcal{L} \sim 100$ fb, as well as a high degree of longitudinal photon polarization can be reached at these colliders.

Tuning the maximum of the $\gamma\gamma$ spectrum to the value of the Higgs boson mass, the Higgs particles can be formed as s–channel resonances, $\gamma\gamma \to$ Higgs, decaying mostly into $b\bar{b}$ pairs. The main background, $\gamma\gamma \to b\bar{b}$, can be suppressed by choosing proper helicities for the initial $e^\pm$ and laser photons which maximizes the signal cross section, and eliminating the gluon radiation by taking into account only two–jet events. Clear signals can be obtained which allow the measurement of the Higgs couplings to photons, which are mediated by loops possibly involving new particles. In addition, in the MSSM, $\gamma\gamma$ colliders allow to extend the reach for the heavy H, A bosons compared to the e^+e^- option.

4.3 Determination of the properties of a SM–like Higgs boson

Once the Higgs boson is found it will be of great importance to explore all its fundamental properties. This can be done at great details in the clean environment of e^+e^- linear colliders [9]: the Higgs mass, the spin and parity quantum numbers and the couplings to fermions, gauge bosons and the self–couplings can measured. Some precision measurements, in particular for the mass and width, can also be performed at the LHC with high–luminosity [8].

In the following we will summarize these features in the case of the SM Higgs boson; some of this discussion can be of course extended to the the lightest MSSM Higgs particle.

4.3.1 Studies at e^+e^- Colliders

• The measurement of the recoil e^+e^- or $\mu^+\mu^-$ mass in the Higgs–strahlung process, $e^+e^- \to ZH \to He^+e^-$ and $H\mu^+\mu^-$, allows a very good determination of the Higgs boson mass. At $\sqrt{s} = 350$ GeV and with a luminosity of $\int \mathcal{L} = 500$ fb^{-1}, a precision of $\Delta M_H \sim 70$ MeV can be reached for a Higgs boson mass of $M_H \sim 120$ GeV. The precision can be increased to $\Delta M_H \sim 40$ MeV by using in addition the hadronic decays of the Z boson [which have more statistics]. Accuracies of the order of $\Delta M_H \sim 80$ MeV can also be reached for $M_H = 150$ and 180 GeV when the Higgs decays mostly into gauge bosons. This one per mile accuracy on M_H can be very important, especially in the MSSM where it allow to strongly constrain the other parameters of the model.

• The angular distribution of the Z/H in the Higgs–strahlung process is sensitive to the spin–zero of the Higgs particle: at high–energies the Z is longitudinally polarized and the distribution follows the $\sim \sin^2\theta$ law which unambiguously characterizes the production of a $J^P = 0^+$ particle. The spin–parity quantum numbers of the Higgs bosons can also be checked experimentally by looking at correlations in the production $e^+e^- \to HZ \to 4f$ or decay $H \to WW^* \to 4f$ processes, as well as in the channel $H \to \tau^+\tau^-$ for $M_H \lesssim 140$ GeV. An unambiguous test of the CP nature of the Higgs bosons can be made in the process $e^+e^- \to t\bar{t}H$ or at laser photon colliders in the loop induced process $\gamma\gamma \to H$.

• The masses of the gauge bosons are generated through the Higgs mechanism and the Higgs couplings to these particles are proportional to their masses. This fundamental prediction has to be verified experimentally. The Higgs couplings to ZZ/WW bosons can be directly determined by measuring the production cross sections in the bremsstrahlung and the fusion processes. In the $e^+e^- \to H\ell^+\ell^-$ and $H\nu\bar{\nu}$ processes, the total cross section can be measured with a precision less than $\sim 3\%$ at $\sqrt{s} \sim 500$ GeV and with $\int \mathcal{L} = 500$ fb^{-1}. This leads to an accuracy of $\lesssim 1.5\%$ on the HVV couplings.

• The measurement of the branching ratios of the Higgs boson are of utmost importance. For Higgs masses below $M_H \lesssim 130$ GeV a large variety of ratios can be measured at the linear collider. The $b\bar{b}, c\bar{c}$ and $\tau^+\tau^-$ branching ratios allow to measure the relative couplings of the Higgs to these fermions and to check the fundamental prediction of the Higgs mechanism that they are proportional to fermion masses. In particular BR$(H \to \tau^+\tau^-) \sim m_\tau^2/3\bar{m}_b^2$ allows to make such a test. In addition, these branching ratios, if measured with enough accuracy, could allow to distinguish a Higgs boson in the SM from its possible extensions. The gluonic branching ratio is sensitive to the $t\bar{t}H$ Yukawa coupling [and might therefore give an indirect measurement of this important coupling] and to new strongly interacting particles which couple to the Higgs [such as stop in SUSY extensions of the SM]. The branching ratio into W bosons starts to be significant for Higgs masses of the order of 120 GeV and allows to measure the HWW coupling. The branching ratio of the loop induced $\gamma\gamma$ decay is also very important since it is sensitive to new particles [the measurement of this ratio gives the same information as the measurement of the cross section for Higgs boson production at $\gamma\gamma$ colliders].

• The Higgs coupling to top quarks, which is the largest coupling in the SM, is directly accessible in the process where the Higgs boson is radiated off top quarks, $e^+e^- \to t\bar{t}H$. For $M_H \lesssim 130$ GeV, the Yukawa coupling can be measured with a precision of less than 5% at $\sqrt{s} \sim 800$ GeV with a luminosity $\int \mathcal{L} \sim 1$ ab^{-1}. For $M_H \gtrsim 350$ GeV, the $Ht\bar{t}$ coupling can be derived by measuring the $H \to t\bar{t}$ branching ratio at higher energies.

• The total width of the Higgs boson, for masses less than ~ 200 GeV, is so small that it cannot be resolved experimentally. However, the measurement of BR$(H \to WW)$ allows an indirect determination of Γ_H since the HWW coupling can be determined from the measurement of the Higgs production cross section in the WW fusion process [or from the measurement of the cross section of the Higgs–strahlung process, assuming SU(2) invariance]. The accuracy of the Γ_H measurement follows then from that of the WW branching ratio. [$\Gamma_{\rm tot}$ can also be measured by using the processes $H \leftrightarrow \gamma\gamma$].

• Finally, the measurement of the trilinear Higgs self–coupling, which is the first non–trivial test of the Higgs potential, is accessible in the double Higgs production processes $e^+e^- \to ZHH$ [and in the $e^+e^- \to \nu\bar{\nu}HH$ process at high energies]. Despite its smallness, the cross sections can be determined with an accuracy of the order of 20% at a 500 GeV collider if a high luminosity, $\int \mathcal{L} \sim 1$ ab^{-1}, is available. This would allow the measurement of the trilinear Higgs self–coupling with an accuracy of the same order.

An illustration of the experimental accuracies which can be achieved in the determination of the mass, CP–nature, total decay width and the various couplings of the Higgs boson for $M_H = 120$ and 140 GeV is shown in Table 1 for $\sqrt{s} = 350$ GeV (for M_H and the CP nature) and 500 GeV (for $\Gamma_{\rm tot}$ and all couplings except for g_{Htt}) and $\int \mathcal{L} = 500$ fb^{-1} (except for g_{Htt} where $\sqrt{s} = 1$ TeV and $\int \mathcal{L} = 1$ ab^{-1} are assumed). For the Higgs self–couplings, the error is only on the determination of the cross section, leading to an error slightly larger, $\sim 30\%$, on the coupling. For the test of the CP nature of the Higgs boson, ΔCP represents the relative deviation from the 0^{++} case.

As can be seen, an e^+e^- linear collider with a high–luminosity is a very high precision machine in the context of Higgs physics. This precision would allow the determination of the complete profile of the SM Higgs boson, in particular if its mass is smaller than ~ 140 GeV. It would also allow to distinguish the SM Higgs particle from the lighter MSSM h boson up to very high values of the pseudoscalar Higgs boson mass, $M_A \sim \mathcal{O}(1$ TeV$)$.

Table 2: Relative accuracies (in %) on Higgs boson mass, width and couplings obtained at TESLA with $\sqrt{s} = 350, 500$ GeV and $\int \mathcal{L} = 500$ fb^{-1} (except for top).

M_H (GeV)	ΔM_H	ΔCP	$\Gamma_{\rm tot}$	g_{HWW}	g_{HZZ}	g_{Htt}	g_{Hbb}	g_{Hcc}	$g_{H\tau\tau}$	g_{HHH}
120	± 0.033	± 3.8	± 6.1	± 1.2	± 1.2	± 3.0	± 2.2	± 3.7	± 3.3	± 17
140	± 0.05	–	± 4.5	± 2.0	± 1.3	± 6.1	± 2.2	± 10	± 4.8	± 23

This is exemplified in Fig. 13, where the (g_{Hbb}, g_{HWW}) and $(g_{Hbb}, g_{H\tau\tau})$ contours are shown for a Higgs boson mass $M_H = 120$ GeV produced and measured at a 500 GeV collider with $\int \mathcal{L} = 500$ fb^{-1}. These plots are obtained from a global fit which take into account the experimental correlation between various measurements. In addition to the 1σ and 2σ confidence level contours for the fitted values of the pairs of ratios, the expected value predicted in the MSSM for a given range of M_A is shown.

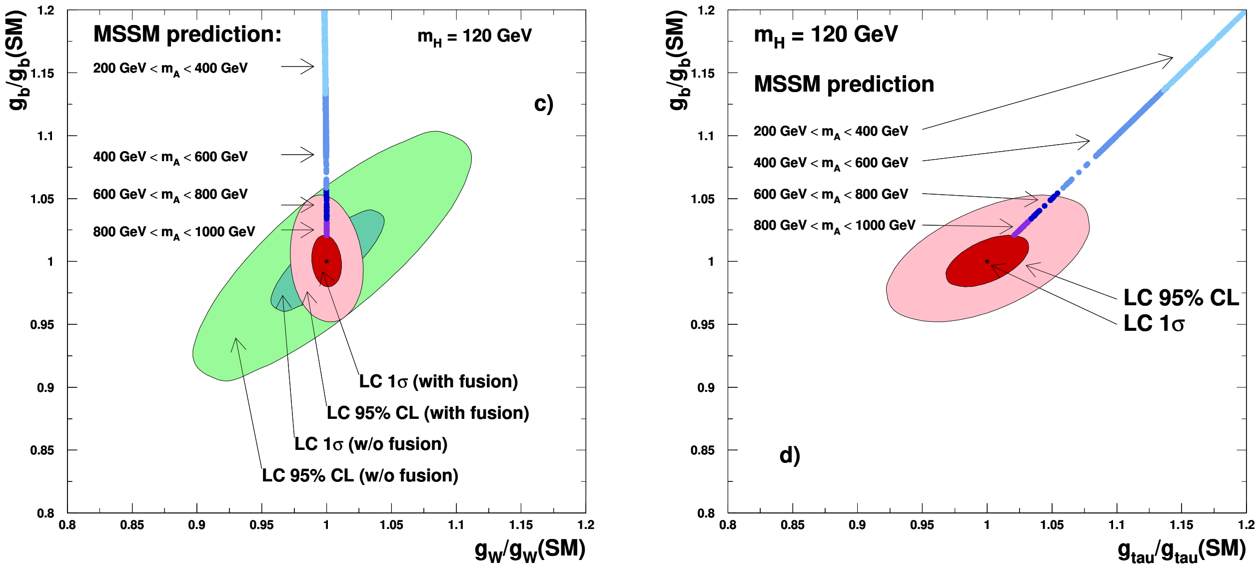


Figure 13: Higgs boson coupling determinations at TESLA for $M_H = 120$ GeV with 500 fb^{-1} of data, and the expected deviations in the MSSM.

4.3.2 Measurements at the LHC

• At the LHC, the Higgs boson mass can be measured with a very good accuracy. In the range below $M_H \lesssim 400$ GeV, where the total width is not too large, a relative precision of $\Delta M_H/M_H \sim 0.1\%$ can be achieved in the channel $H \to ZZ^{(*)} \to 4\ell^\pm$ with 300 fb^{-1} luminosity. In the 'low mass" range, a slight improvement can be obtained by considering $H \to \gamma\gamma$. In the range $M_H \gtrsim 400$ GeV, the precision starts to deteriorate because of the smaller cross sections. However a precision of the order of 1% can still be obtained for $M_H \sim 800$ GeV if theoretical errors, such as width effects, are not taken into account.

• Using the same process, $H \to ZZ^{(*)} \to 4\ell^{\pm}$, the total Higgs width can be measured for masses above $M_H \gtrsim 200$ GeV when it is large enough. While the precision is very poor near this mass value [a factor of two], it improves to reach the level of $\sim 5\%$ around $M_H \sim 400$ GeV. Here, again the theoretical errors are not taken into account.

• The Higgs boson spin can be measured by looking at angular correlations between the fermions in the final states in $H \to VV \to 4f$ as in e^+e^- collisions. However the cross sections are rather small and the environment too difficult. Only the measurement of the decay planes of the two Z bosons decaying into four leptons seems promising.

• The direct measurement of the Higgs couplings to gauge bosons and fermions is possible but with a rather poor accuracy. This is due to the limited statistics, the large backgrounds and the theoretical uncertainties from the limited precision on the parton densities and the higher order radiative corrections or scale dependence. An example of determination of cross sections times branching in various channels at the LHC is shown in Fig. 14 for a luminosity of 200 fb^{-1}. Solid lines are for gg fusion, dotted lines are for $t\bar{t}H$ associated production with $H \to b\bar{b}$ and WW and dashed lines are the expectations for the weak boson fusion process. A precision of the order of 10 to 20% can be achieved in some channels, while the vector boson fusion process leads to accuracies of the order of a few percent. However, experimental analyses accounting for the backgrounds and for the detector efficiencies as well as further theory studies for the signal and backgrounds need to be performed to confirm these values. The Higgs boson self–couplings are too difficult to measure at the LHC because of the smallness of the $gg \to HH$ and $qq \to HHZ$ cross sections and the large backgrounds.

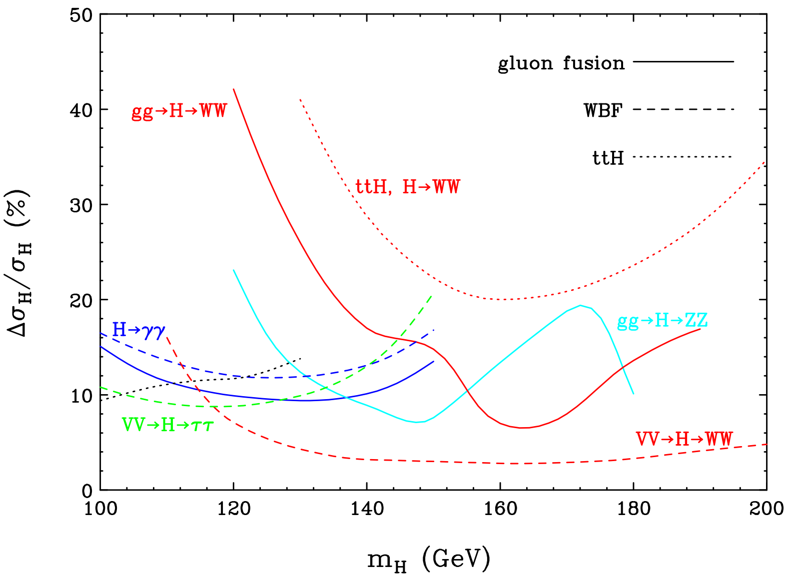


Figure 14: Expected relative errors on the determination of $\sigma \times$ BR for various Higgs boson search channels at the LHC with 200 fb^{-1} data.

• To reduce the theoretical uncertainties and some experimental errors, it is more interesting to measure ratios of quantities where the overall normalization cancels out. For instance, by using the same production channel and two decay modes, the theory error from higher order corrections and from the poor knowledge of the parton densities drops out in the ratios. A few examples of measurements of ratios of branching ratios or ratios of Higgs couplings squared at the LHC with a luminosity of 300 fb^{-1} are shown in Table 2 [with still some theory errors not included in some cases]. As can be seen, a precision of the order of 10 percent can be reached in these preliminary analyses.

Table 3: Relative accuracies on measurements of ratios of cross sections and/or branching ratios at the LHC with a luminosity of 300 fb^{-1}.

Process	Measurement quantity	Error	Mass range
$\frac{(t\bar{t}H+WH)\to\gamma\gamma+X}{(t\bar{t}H+WH)\to b\bar{b}+X}$	$\frac{\mathrm{BR}(H\to\gamma\gamma)}{\mathrm{BR}(H\to b\bar{b})}$	$\sim 15\%$	80 – 120 GeV
$\frac{H\to\gamma\gamma}{H\to 4\ell^+}$	$\frac{\mathrm{BR}(H\to\gamma\gamma)}{\mathrm{BR}(H\to ZZ^*)}$	$\sim 7\%$	120 – 150 GeV
$\frac{t\bar{t}H\to\gamma\gamma,b\bar{b}}{WH\to\gamma\gamma,b\bar{b}}$	$\left(\frac{g_{Htt}}{g_{HWW}}\right)^2$	$\sim 15\%$	80 – 120 GeV
$\frac{H\to ZZ^*\to 4\ell^+}{H\to WW^*\to 2\ell^\pm 2\nu}$	$\left(\frac{g_{HZZ}}{g_{HWW}}\right)^2$	$\sim 10\%$	130 – 190 GeV

A more promising channel would be the vector boson fusion process, $qq \to WW/ZZ \to Hqq$ in which $H \to \tau^+\tau^-$ or $WW^{(*)}$, which would allow the additional measurement of the couplings to τ leptons for instance. A preliminary parton level analysis including this production channel shows that measurements of some Higgs boson couplings can be made at the level of 5–10% statistical error. More work, including full detector simulation, is needed to sharpen these analyses.

4.3.3 Complementarity between the LHC and LC

In many aspects, the searches at e^+e^- colliders are complementary to those which will be performed at the LHC. An example can be given in the context of the MSSM. In constrained scenarios, such as the minimal Supergravity model, the heavier H, A and $H^\pm$ bosons tend to have masses of the order of 1 TeV and therefore will escape detection at both the LHC and linear collider. The right–handed panel of Fig. 15 shows the number of Higgs particles in the $(M_A, \tan\beta)$ plane which can observed at the LHC and in the white area, only the lightest h boson can be observed. In this parameter range, the h boson couplings to fermions and gauge bosons will be almost SM–like and, because of the relatively poor accuracy of the measurements at the LHC, it would be difficult to resolve between the SM and MSSM (or extended) scenarii. At e^+e^- colliders such as TESLA, the Higgs couplings can be measured with a great accuracy, allowing to distinguish between the SM and the MSSM Higgs boson close to the decoupling limit, i.e. for pseudoscalar boson masses which are not accessible at the LHC. This is exemplified in the right-panel of Fig. 15, where the accuracy in the determination of the Higgs couplings to $t\bar{t}$ and WW are displayed, together with the predicted values in the MSSM for different values of M_A. The two scenarii can be distinguished for pseudoscalar Higgs masses up to 1 TeV.

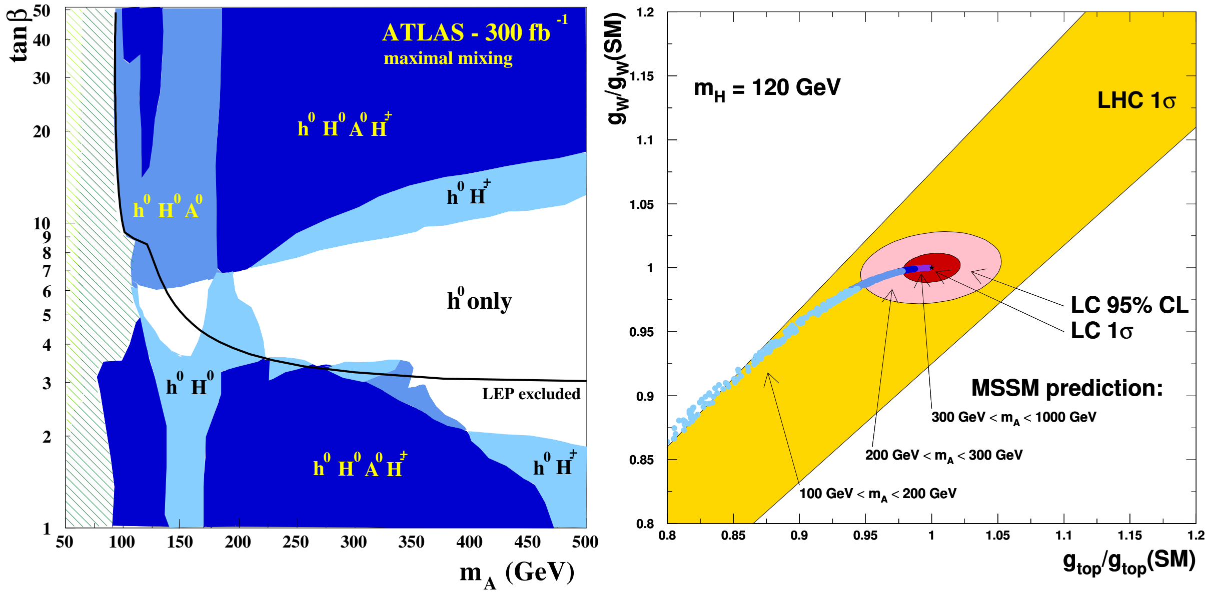


Figure 15: Number of Higgs bosons which can be observed at the LHC in the $(M_A, \tan\beta)$ plane (right) and a comparison of the accuracy in the determination of the g_{ttH} and g_{WWH} couplings at the LHC and at TESLA compared to the predictions from MSSM for different values of M_A.

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